Comparative analysis of economic consequences of pension tax systems: EET vs. TEE in the Netherlands amid demographic changes

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Abstract

This thesis compares the economic consequences of EET and TEE pension tax systems in the Netherlands and their response to demographic change using a general equilibrium overlapping generations (OLG) model constructed and calibrated to the Dutch economy. The OLG model incorporates occupational pension schemes with mandatory pension contributions, an important factor in the Dutch pension provision. Findings indicate that TEE yields higher economic output compared to EET. Moreover, higher subsidy levels under TEE, where the government contributes to pension savings as a percentage of the individual's savings, result in higher economic output. Additionally, the study suggests that when mandating self-employed to save, the economic output under EET is higher than under TEE. Demographic scenarios show that the economic output under EET and TEE react similarly to demographic changes. Moreover, the study can not confirm that tax revenue and state pension expenditures move better along under EET compared to under TEE when the dependency ratio declines, as suggested in literature. The sensitivity analysis indicates the dependency of the conclusion, that TEE yields a higher economic output, to underlying model assumptions. Nevertheless, individual welfare, as measured by the average certainty equivalent, is consistently higher under EET in all investigated scenarios.

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1 Introduction

The Netherlands has the largest amount of pension assets expressed as a percentage of GDP worldwide (Thinking Ahead Institute, 2023). Like most OECD countries, it applies a variant of the "Exempt-Exempt-Taxed" (EET) tax system to retirement savings.[1] Here, both pension contributions and returns on investment are exempted from taxation, while benefits are treated as taxable income upon withdrawal (OECD, 2022). The tax payment is therefore under the EET tax system delayed. Given the amount of pension capital in the Netherlands, one can imagine the large amount of deferred tax payments included in the pension capital.

Pension taxation methods vary across the globe. Pension saving involves three transactions, each presenting an opportunity for tax collection: (i) when saving part of the income, (ii) when investment income and capital gains accumulate, and (iii) when receiving pension benefits. If pensions are pay-as-you-go financed, the opportunity to tax at the second point is lost. A triplet of letters describes a basic tax policy given the three tax collection points. Both the state pension, which is in the Netherlands pay-as-you-go financed, and private pensions are subject to an EET system in the Netherlands, exempting the first two stages from taxes (E) and taxing the third (T). Only for incomes exceeding €137,800 in 2024, a TEE system applies to the private scheme in the Netherlands (Belastingdienst, 2024).

Despite the order of magnitude of tax pension payments, there is limited literature on capturing the effects of different pension tax payment systems on the economy. The limited literature suggests the near equivalence between using EET or TEE. When the tax rates valid during working life and retirement are equal, the EET and TEE systems result in equal tax benefits (Huang, 2008). Romaniuk (2013) confirms that with a constant tax rate and initial contribution, the tax systems are equivalent in terms of risk-taking.

However, assuming equal tax rates during working life and retirement is unrealistic. In many countries, like the Netherlands, the marginal tax during working life is higher than during retirement (Ministerie van Financiën, 2020). When considering this difference in income tax rates, Romaniuk (2013) shows that the near equivalence for the EET and TEE tax systems breaks down when considering risk-taking by asset-liability management of pension funds. She proves that the asset-liability management of DC funds is risk-taking neutral under the TEE system, while the EET system can affect risk-taking behaviour (Romaniuk, 2013). The difference is created by the occurrence of the pension tax rate in the optimisation problem of the asset manager under the EET tax system, in contrast to the TEE tax system. Risk aversion properties drive the direction of this tax effect (Romaniuk, 2013). Moreover, Armstrong et al. (2015) uses an overlapping generation model to show that EET generally results in higher pension savings in a country with a progressive taxation system and a flexible labour supply. TEE leads to front-load taxation onto younger households, reducing their available resources. Consequently, young households decrease their consumption and savings. The decreased savings, and therefore the investment, reduces the capital available to the economy, reducing productivity, the economy's output and wages (Armstrong et al., 2015).

[1] OECD is an abbreviation for Office of Community Economic Development, an intergovernmental organisation with 28 developed member countries.

The Netherlands introduced the EET tax system on 1 June 1999, among other rules regarding pensions written in the "Witteveenkader". Taxing pension benefits and making the pension contributions deductible from taxable income is called "omkeerregeling" in Dutch. Implementing the EET system initially declined tax revenue, compensated by taking extra public debt (Beetsma et al., 2011). Besseling (1998) notes that as a consequence of the EET taxation, the government participates in the risks the asset management takes and, thus, the return achieved. The return on the tax deferral was expected to be higher than the interest expense on the additional debt due to the tax deferral (Besseling, 1998).

Due to the progressive tax system, the lower tax rate in the first income tax bracket after passing the state pension age, and the absence of taxation on the accrual of assets in the accumulation phase, the "omkeerregel" is often seen as a subsidy to encourage pension savings. This implication serves well as it is commonly known from behavioural economics that many people lack the willpower to save for later (Starink et al., 2016). However, Starink et al. (2016) suggest that this effect is absent in the Netherlands because the vast majority of the working population is compulsorily saving for pension. Also Don et al. (2013) conclude that the fiscal facilitation in the EET regime goes along with "dead-weight-loss": also without this fiscal facilitation people would still build pensions through mandatory workplace arrangements. Moreover, Don et al. (2013) question the effectiveness of the fiscal facilitation to stimulate pension savings as the savings of self-employed individuals without workplace pension arrangements are limited, even though they benefit from the fiscal facilitation. Research on the so-called "401(k) pension plans" in the US, where employers subsidise employees' pension contributions up to a certain limit, support the idea that subsidising retirement savings has limited effectiveness (Choi et al., 2006). Nevertheless, Don et al. (2013) recognise that fiscal facilitation increases support for mandating pension accrual among employees and recognises that without likely less ambitious are created.

Don et al. (2013) also mention some indirect consequences of the EET tax system. Don et al. (2013) suggest that tax facilitation affects people's investment choices because retirement investment is more attractive than paying off their mortgage or investment in education. Moreover, high-income earners benefit slightly more from the tax facilitation due to the progressive tax system. Lastly, Don et al. (2013) recognise how the EET tax system protects the government as tax revenues from the second pillar move better along with the amount of public pensions paid.

In an opinion piece, de Vos (2021) claims that the deferred tax amounts to a minimum of 33% of the pension assets, totalling 619 billion euros. This could be collected by moving to the TEE tax system and introducing a 33% tax rate on pension contributions, retroactively applied to the entire pension portfolio. Future pension payouts would not be taxed anymore. With an equal tax rate at working age and retirement, pension funds would maintain their funding ratio and for the participants, it would not make a net difference. De Vos 2021 notices that collecting all deferred taxes at once could be disruptive and suggests it be done gradually, in a period of, for example, 11 years. [2]

[2] Although the total pension funds declined after the opinion piece of De Vos 2021 to an amount of €1,542 billion in the first quarter of 2023, using the same tax rate, the change in tax policy would still yield 509 billion euros in tax revenue (De Nederlandsche Bank, 2023b).

De Vos 2021 does not discuss the economic consequences of such an implementation of the TEE tax system. This thesis explores the economic implications of the currently implemented Exempt-Exempt-Taxed tax system (EET) and the Taxed-Exempt-Exempt tax system (TEE) in the context of the Netherlands. In the remainder of this thesis, I abbreviate the Exempt-Exempt-Taxed tax system as EET and the Taxed-Exempt-Exempt tax system as TEE. I delve into the distinctions rather than investigating the effect of altering the tax system. Examining such effects would introduce various aspects, including political debate, participant behaviour in response to change, and legal aspects.

My research question is: *What are the consequences of the Exempt-Exempt-Taxes (EET) and the Taxed-Exempt-Exempt (TEE) pension tax systems on the economy and welfare across generations and income groups amidst demographic changes in the context of the Netherlands?*

I use an Auerbach-Kotlikoff Overlapping Generations (OLG) Model, a framework that has become a widely used instrument in the quantitative analysis of public matters related to intergenerational redistribution. The economy comprises households, firms, and the government (Auerbach and Kotlikoff, 1987). The OLG model I use is an adaptation of the model used by Armstrong et al. (2015), customized to represent the pension system of the Netherlands, with demographics and parameters calibrated to the Dutch economy. Moreover, I alter the calculation of the economic output, and the government constraint under EET used by Armstrong et al. (2015) to ensure accuracy. Until now, to the best of my knowledge, the effects of the different tax regimes on the Dutch economy are unexplored. The main contribution to related literature is the introduction of occupational pension schemes with mandatory pension contributions, an important factor in the Dutch pension provision. Introducing mandatory pension savings can have a major impact on the results, and as far as my knowledge goes, it has not been researched yet. Moreover, I investigate the economic impact of demographic changes, driven by fertility and longevity, on the different tax systems. As the Netherlands witnesses an ageing population and changing fertility rates, the ability of tax systems to adapt to these changes becomes more important.

The results reveal that the economic output under TEE is higher than under EET, with higher subsidy levels, as a percentage of the individuals' savings paid by the government, increasing the economic output under TEE. The decrease in capital under TEE compared to EET resulting from the elimination of the delayed taxes, is compensated by a substantial increase in labour under TEE. However, the sensitivity analysis shows that the conclusion, that the economic output is higher under TEE than under EET, depends on underlying assumptions. Moreover, with a legislative change requiring self-employed individuals to save through workplace pension arrangements, the economic output under EET is higher than under TEE, with a pension subsidy of 20%. Under all scenarios considered, TEE reduces the average economic well-being level of the population, calculated as the average certainty equivalent. Particularly, the highest-earning employed and self-employed individuals are affected. Demographic scenarios show that the economy's output under EET and TEE react similarly to demographic changes with nuances on the impact of the demographic change on the components of the output. The study cannot confirm that EET provides better protection for the government and future generations against variations in the dependency ratio, as the tax revenue does not move better along with the state pension expenditures under EET than under TEE.

This thesis is organised as follows. In Section 2, I introduce the overlapping generations model with the EET tax system, designed to correspond to the EET tax system currently implemented in the Netherlands, and elaborate on how the model changes when implementing TEE. In Section 3, I calibrate the model's parameters to the Dutch economy seen in 2019. Section 4 elaborates on the model's validity. In Section 5.1, the results of the EET and TEE tax schemes are presented, the last with multiple levels of subsidies to compensate for the disadvantage of higher taxes at working life than at retirement. In Section 5.2, I report the impact of demographic changes on the economy under the pension tax systems. In Section 6, I investigate the model's sensitivity to underlying assumptions. Section 7 concludes, and in Section 8, I reflect on the meaning and implication of the results, but also the study's limitations.

2 The overlapping generations model representing the Dutch economy

The National Institute of Economic and Social Research commissioned Armstrong et al. (2015) to examine the economic implications of transitioning from an EET to a TEE pension tax system in the United Kingdom. To assess the economic consequences, Armstrong et al. (2015) use a general equilibrium overlapping generations (OLG) model consisting of households, firms and the government in a closed-economy setting with no international trade. The model features markets for goods, labour and capital, with market clearing conditions for labour and capital. The goods market clearing condition is redundant by Walras' law.

In the following Section, I describe the OLG model used and explicitly explain how I change, or what I add to, the model of Armstrong et al. (2015). The main additions or changes are: (i) I adjust the pension structure and tax system to align with the present conditions in the Netherlands. For example, I distinguish between employees who make mandatory pension savings through workplace arrangements and self-employed, who have the freedom to choose their pension contribution. (ii) Agents include the probability of survival in their optimisation problem. (iii) I introduce demographics in the model. With this introduction, I can investigate the impact of demographic changes on the economy under both tax tax systems. (iv) I calculate the output under both EET and TEE with an adjustment to ensure accuracy. (v) I adjust to government constraint under EET to ensure accuracy. Moreover, (vi) the tax income of the agents is not fixed and is dependent on the agents' income. Lastly, (vii), I reallocate accrued pension capital among survival peers in the case of death.

First, I describe the Dutch pension system and its application in the model in Section 2.1. I continue with describing the demographics and intra-cohort heterogeneity observed in the model in Section 2.2. Section 2.3, I divide into two sections, each describing the optimisation problem of the households under both tax systems. Section 2.4 outlines the market clearance conditions. I proceed in Section 2.5 to expain on the optimisation problem firms face, and Section 2.6 details the government's budget constraint. Finally, in Section 2.7, I clarify the simulation methods used.

2.1 Application Dutch pension system in the model

The Dutch pension system consists of three pillars: the state pension (AOW), the pension employees build up through workplace arrangements and voluntary individual income provisions. In this Section, I briefly introduce the pillars seen in the Netherlands and make some remarks regarding their application in the model.

1. State pension. The so-called "Algemene Ouderdomswet" (AOW) benefits are the state pension everyone receives from the government when reaching the national retirement age. The AOW is a basic provision at the minimum level and is often supplied with pensions accrued from the second and third pillars. The state pension is funded from tax revenues and not explicitly from the tax paid by the recipient: therefore it is a pay-as-you-go system (Pensioen Federatie, 2019).

A resident of the Netherlands is eligible for a full state pension when he or she has lived in the country for 50 years. All individuals in the model are assumed to have spent their entire working lives in the country, and subsequently, they receive full benefits. Furthermore in the model, similar to reality, the benefits are subject to taxation when receiving the state pension.

2. Workplace pension. The second pillar consists of the occupational pensions accrued by the vast majority of employees during their working lives. These employee pensions must be held outside of the company of the employer.[3] The pension funds, where the pension contributions are stored, are based on capital funding. The employers and employees contribute to agreed collective employment agreements, which are subsequently invested in, for example, equity, real estate and bonds. The accumulated assets are used to pay out pension benefits in the future (Pensioen Federatie, 2019). In this thesis, the employee makes all contributions. Although in reality, not all employees participate in workplace arrangements and occupational pension schemes often differ industry-wide, I assume all employees participate in the same workplace arrangement.

Currently, a defined benefits (DB) plan is implemented in the Netherlands. Here, the pension plan guarantees a specified payment amount at retirement and the contribution rate adjusts to meet this criteria. The pension contribution is determined based on an accrual rate of the final pension, which is a complex calculation involving so-called actuarial factors. As these calculations introduce much complexity, I determine the pension contribution as a percentage of the so-called pensionable base. The pensionable base is the salary minus the so-called franchise. The franchise is introduced because the workplace pension is additional to the state pension (AOW), and therefore one does not have to accrue pension over the first part of his salary. Calculating the pension contribution as a percentage of the pensionable base aligns with the proposed new pension system, "the Wet Toekomst Pensioenen", which must be implemented at all pension funds no later than January 1, 2028. In the OLG model, the second pillar is modelled as a funded defined benefits plan (DC), how it is proposed in the new pension system.

Many occupational pension schemes include survivor benefits. Then, if a participant passes away, a portion of the accrued pension benefits are paid to the surviving spouse

[3] I assume that the probability of the particular pension fund, in which the savings are held, to default is zero.

or partner. The rest of the accrued pension benefits are added to the collective pension assets. I do not include partner- and/or orphan pensions in the model, therefore all accrued pension benefits are divided among survival peers (of the same age and income class) in the event of death.

3. Private pensions. In addition to the workplace pension, one can also make voluntary contributions to a private pension fund. For individuals without workplace pensions, who, for example, own their own business, this is the only way to build a pension along with the state pension.

Similar to reality, pensions accrued from the second and third pillars are taxed via the EET system. There is a ceiling from which one can not accrue pension benefits from tax-free contributions. I do not include this ceiling as the highest salary in my model is below the ceiling.

2.2 Demographics and agent heterogeneity

In this Section, I introduce the demographics and intracohort heterogeneity seen in the OLG model. Time is discrete ($t = 0, 1, 2, 3, ..$), and the economy lives forever. The subscript t denotes the current period, and a period represents 20 years. Put differently, a leap in time from t to $t + 1$ represents 20 years. The population is divided into three age groups (20-40, 40-60 and 60-80). Thus, I simplify the model by assuming the absence of children based on the notion that individuals become economically valuable when they start working at the age of 20. Every period, a cohort of individuals join the workforce (age group 20-40). Each individual can live for a maximum of 3 periods. The individual works the first two periods, after which the individual retires in the third period. In the model, I do not include unemployment or disability risk. Hereafter, for simplicity, I call these age groups the young adult group, the middle-age group and the retired, respectively. I denote these groups in mathematical abbreviations as $s = 1$, $s = 2$ and $s = 3$. Moreover, $pop_{s,t}$ denotes the number of individuals in a particular age group s at time t. In the remainder, I call these individuals agents.

Armstrong et al. (2015) implicitly assume all age groups to be equally large. In contrast to Armstrong et al. (2015), I investigate the effects of changing demographics under the EET and TEE tax systems. In my model, the size of the age groups can fluctuate over time due to changes in survival rates, fertility and migration, denoted by $\psi_{t,s}$, fr_t and mr_t, respectively. The survival rate, $\psi_{t,s}$, is the probability of an age group s at time t to reach the age group $s + 1$ at time $t + 1$. The individual lifespan uncertainty is dependent on time and age and independent of an agent's productivity level. I assume that every newborn child ($s = 0$) turns 20 with probability 1 and therefore $\psi_{0,t} = 1$. The fertility rate, fr_t, is one if every couple has two children. In this case, the population stays the same without a change in ageing.

The size of the age groups changes over time due to changes in fertility and ageing. I measure the size of the age groups as follows:

$$pop_{s,t} = \begin{cases} pop_{1,t-1} fr_{t-1} + mr_t & \text{for } s = 1 \\ pop_{s-1,t-1} \psi_{s-1,t-1} & \text{for } s \in \{2, 3\} \\ 0 & \text{for } s > 3 \end{cases} \tag{1}$$

The first equation captures both the effects of fertility in the country and the effects of migration. I assume that couples in the age group 20-40 start a family, and one period (20 years) later, the child joins the workforce at age 20. The first equation thus implies that the number of children born at time t depends on the size of the cohort of the first adult group (20-40 years) at time $t-1$ multiplied by the fertility rate at time $t-1$. I add the number of native newborn babies to the number of net migrants at time t, mr_t. For simplicity, I assume all migrants enter the economy at age 20. Therefore, net migration does not appear in the second equation.[4] I assume migrants possess the same skill composition as natives.

The second equation captures the effects of ageing. The size of age group s ($s \neq 1$) at time t is the cohort size of age group $s-1$ at time $t-1$ multiplied by the probability of age group $s-1$ at $t-1$ to survive to age group s at time t. When there is ageing among the population, the survival rate increases over time t. The third equation reflects the assumption that an agent dies with probability one at 80 years old, implying that $\psi_{3,t} = 0$.

Also within an age group there is heterogeneity. I distinguish three different types of taxpayers in an age group: those subject to the basic rate, higher rate and additional rate of income taxation. I call these the basic, middle and highest-earning agents, denoted henceforth in abbreviations as $j = 1$, $j = 2$ and $j = 3$ respectively. The three tax rates correspond to a specific tax bracket. The productivity of the agent type j, ρ_j, is calibrated such that his average wage during working life matches the average salary in tax bracket j. The share of each type of agent/taxpayer among the population is denoted by π_1, π_2, and π_3. These shares are assumed to be the same across all age groups and stay constant over time. Moreover, I assume that the productivity of an individual does not evolve over time. The individual has correct projections on this matter and thus assumes that his or her productivity stays the same in both working periods.

The total population at time t can be represented by:

$$pop_t = \sum_{s=1}^{3} \sum_{j=1}^{3} \pi_j pop_{s,t} \tag{2}$$

Also between the agents of type j I make a discrepancy. In contrast to the model of Armstrong et al. (2015), I distinguish between employees and self-employed. As explained in Section 2.1, a large proportion of the Dutch working population accrues pension through a mandatory pension scheme in the second pillar. Therefore, I divide the population by employees and self-employed with shares denoted by Φ and $(1 - \Phi)$, respectively. The

[4] If the migrant enters the model at a later age, their optimisation problem changes, as they optimise their total utility over a shorter amount of time. Moreover, the migrant will not get full AOW benefits. Additionally, it is challenging to determine savings made before entering the country. All contribute to the decision to make this assumption.

distribution of productivity of employees and self-employed are assumed to be the same. However, as mentioned in Section 2.1, the employee faces more constraints on his pension savings. In the following Section 2.3, I simultaneously outline the optimisation problem of both and how they differ. Here, I do not differ in notation between employees and self-employees. The resulting choices of labour, consumption, and saving of the self-employed and employees are denoted by the superscript 's' and 'e', respectively. Starting from Section 2.5, this difference is essential to ultimately determine the results.

As a last remark, I assume agents to be rational, optimizing explicit objective functions, subject to constraints they face. Moreover, I assume homogeneous preferences among all agents.

2.3 Household optimization problem under the EET and TEE tax systems

Optimisation problem
All agents maximise the discounted sum of lifetime utility they receive from consuming (+) and working (-). In contrast to the model of Armstrong et al. (2015), where agents survive all ages with probability 1, the agents do not survive all ages with certainty, and they take this fact into account in their optimisation problem. Before entering the model, all agents optimise their lifetime utility at time $t-1$. Agents lack the opportunity to reconsider their decision after the realisation of the last period. Agent j maximises the following equation before entering the model at time $t-1$.

$$\max_{\{c_{s,j,t+s-1},l_{s,j,t+s-1}\}_{s=1}^{3}} \sum_{s=1}^{3} \beta^{s-1} u(c_{s,j,t+s-1}, l_{s,j,t+s-1}) \prod_{m=1}^{s} \psi_{m-1,t+m-2} \tag{3}$$

Here, $u(c_{s,j,t}, l_{s,j,t})$ denotes the utility agent j receives from consuming (c) and the discomfort from working (l) in age group s at time t. Recall that $\psi_{s,t}$ is the survival rate at time t for age group s to reach age group $s+1$ at time $t+1$. As mentioned in Section 2.2, I assume that all agents enter the economy at age 20 with probability 1, and therefore $\psi_{0,t} = 1$ at any time t. Like Armstrong et al. (2015), I assume the Cobb-Douglas utility function with multiplicative, non-separable preferences:

$$u(c_t, l_t) = \frac{(c_t^{\gamma}(1-l_t)^{1-\gamma})^{1-\sigma}}{1-\sigma} \tag{4}$$

Here, γ denotes the relative weight for consumption versus leisure. σ accounts for consumption smoothing over time.

The following Table shows all abbreviations used in the optimisation problem and henceforth in the model:

Table 1: Abbreviations description

Parameter	Description
$c_{s,j,t}$	consumption of agent type j in age group s at time t
$s_{s,j,t}$	savings of agent type j in age group s at time t
$l_{s,j,t}$	ratio of working time (/labour supply) to total available time of agent j in age group s at time t
$k_{s,j,t+1}$	pension capital agent type j in age group s, at the beginning of time $t+1$
$\tau_{s,j,t}$	income tax rate of agent type j in age group s at time t
T_s,j,t	lump-sum payment of agent type j in age group s at time t
r_t	real interest rate at time t
w_t	real wage rate at time t
$\tilde{b}$ / f	state pension / franchise
$pop_{s,t}$	size of age group s at time t
$\psi_{s,t}$	probability at time t of age group s to survive age group $s+1$ at time $t+1$ conditional on all information χ at time $t-1$
fr_t	fertility rate at time t
mr_t	net migration rate at time t

Budget constraint working life
Each agent with productivity type j has a budget constraint during working life ($s \in \{1,2\}$) at time t given by:

$$(1+\tau_c)c_{s,j,t} + s_{s,j,t} = w_{j,t}l_{s,j,t} - T(w_{j,t}l_{s,j,t}, s_{s,j,t}) \quad (5)$$

Here, $c_{s,j,t}$ is the consumption of a type j agent in age group s at time t. Furthermore, I denote the pension savings of the type j agent in age group s at time t as $s_{s,j,t}$, which are non-accessible before retirement. w_t is the real wage rate. Moreover, $l_{s,j,t}$ is the labour supply, which is assumed to be zero in the retired period ($l_{3,j,t} = 0$). Then, $w_{j,t}l_{s,j,t}$ is the income of agent type j at time t, where $w_{j,t} = \rho_j w_t$ and ρ_j is the productivity of agent type j. Income tax liabilities $T(w_{j,t}l_{s,j,t}, s_{s,j,t})$ depend upon labour income and possibly on the amount of pension savings in an EET model.

During the working aged periods ($s \in \{1,2\}$), the agent chooses the amount of long-term savings $s_{s,j,t}$. These pension savings accumulate into a pension capital stock $k_{s,j,t+1}$. I assume that the agents enter the economy without capital and therefore $k_{1,j,t} = 0$. For $s \in \{1,2\}$:

$$k_{s+1,j,t+1} = \frac{1}{\psi_{s,t}}(k_{s,j,t}(1 + r_t + \delta) + s_{s,j,t}(1+\tau_b)) \quad (6)$$

Here, δ represents the period-to-period depreciation rate of capital. Additionally, τ_b is a subsidy, which is zero under EET. In line with the proposal of Armstrong et al. (2015), I introduce a subsidy rate τ_b under TEE paid by the governance. τ_b is the same for all agents and serves as a strategic tool to promote retirement savings under the TEE, mitigating the impact of the disadvantage posed by the higher tax rates during working life. As can be seen in equation (6), the subsidy is a percentage of the savings of the agent. In contrast to Armstrong et al. (2015), I use r_t in the calculation instead of r_{t+1} as $k_{s,j,t}$ is measured at the beginning of time t.

Recall that $\psi_{s,t}$ is the probability for an agent in age group s at time t to reach age group $s+1$ at time $t+1$. I add $\frac{1}{\psi_{s,t}}$ to the equation in the model of Armstrong et al. (2015), as the wealth of those who pass away at time t is evenly distributed among their survival peers (of the same age group and same productivity). As mentioned in Section 2.1, this assumption aligns with current regulations in both the second and third pillars. The accuracy of this addition to the is explained in Theorem 2.1:

Theorem 2.1. *When assuming pension capital division among survival peers, of the same age group and income class, in the case of death and the demographics seen as in equation (1). Then the capital accumulation is computed as:*

$$k_{s+1,j,t+1} = \frac{1}{\psi_{s,t}}(k_{s,j,t}(1 + r_t + \delta) + s_{s,j,t}(1 + \tau_b))$$

Proof. To proof Theorem 2.1, one has to show that with no savings made, the net present value of $pop_{s+1,t+1}\pi_j k_{s+1,j,t+1}$ at time t is equal to $pop_{s,t+1}\pi_j k_{s,j,t}$. Therefore:

$$pop_{1,t}\pi_j k_{1,j,t} = pop_{2,t+1}\pi_j \frac{k_{2,j,t+1}}{(1 + r_t + \delta)} = pop_{2,t+1}\pi_j \frac{\frac{1}{\psi_{1,t}} k_{1,j,t}(1 + r_t + \delta)}{(1 + r_t + \delta)} \quad (7)$$

$$pop_{2,t}\pi_j k_{2,j,t} = pop_{3,t+1}\pi_j \frac{k_{3,j,t+1}}{(1 + r_t + \delta)} = pop_{3,t+1}\pi_j \frac{\frac{1}{\psi_{2,t}} k_{2,j,t}(1 + r_t + \delta)}{(1 + r_t + \delta)} \quad (8)$$

Rewriting gives: $pop_{2,t+1} = sr_{1,t}pop_{1,t}$ and $pop_{3,t+1} = sr_{2,t}pop_{2,t}$, equal to the dynamics seen in equation (1). Given the market clearings function (44), the net present value of the pensions value at time t+1, assuming no in-between savings, are equal to the pension assets at time t. □

Thus, when agent type j decides their future consumption and leisure, he takes not only his own expected survival rate into account, but also their peers. The agents assumes these are the same, however, it is fairly simple to change, and it might be interesting as longevity perceptions are often biased (Perlman et al., 2020).

Budget constraint in retirement
Agent j consumes all pension savings and the state pension in retirement. Therefore:

$$s_{3,j,t+2}(1 + \tau_b) = -k_{3,j,t+2}(1 + r_{t+2} + \delta) \quad (9)$$

In retirement, the agents have additional to the accumulated pension capital $k_{3,j,t+2}(1 + r_{t+2} - \delta)$ a tax-financed state pension $\tilde{b}$ they consume. I assume there is no bequest motive. The budget constraint for the retired, $s = 3$, therefore becomes:

$$(1 + \tau_c)c_{3,j,t+2} = k_{3,j,t+2}(1 + r_{t+2} - \delta) + \tilde{b} - T(k_{3,j,t+2}(1 + r_{t+2} - \delta), \tilde{b}) \quad (10)$$

State pensions are taxed at retirement, aligning with present circumstances in the Netherlands (Ministerie van Financiën, 2020). In contrast to the model of Armstrong et al. (2015), where the state pension is tax-free.

Constraints on pension savings
In the model, I distinguish between employees and retirees. Employees save mandatory workplace pensions in the second pillar as described in Section 2.1. They can accrue additional pension entitlements in the third pillar by making voluntary contributions to private pension schemes. Self-employees, however, can only make voluntary pension contributions to private pension funds in pillar three.

Employees pay mandatory contributions in the second pillar: a certain percentage, denoted by ϕ, of their pensionable base. As explained in Section 2.1, the pensionable base is the salary minus the franchise. All savings above the mandatory savings are contributions to a private pension fund in the third pillar. Under TEE, I adjust the percentage of mandatory savings as the savings in TEE are not tax exempted like under EET, while the pension benefits are. Moreover, for both employees and self-employed, I assume pension savings are non-accessible for all agents in the first two periods.

In the following Section, I elaborate on the two models for pension taxation: one follows an EET tax system, and the other adopts a TEE tax system.

2.3.1 Exempt-Exempt-Taxed (EET) pension tax system

Under the EET regime, pension savings $s_{s,j,t}$ can be deducted from taxable income. Keeping the model sketched in the previous Section in mind, the agent's budget constraint for the young adult group becomes:

$$(1+\tau_c)c_{1,j,t} + s_{1,j,t} = w_{j,t}l_{1,j,t} - \tau_{1,j,t}(w_{j,t}l_{1,j,t} - s_{1,j,t}) + T_{1,j,t} \tag{11}$$

Likewise, the budget constraint for the middle-age group becomes:

$$(1+\tau_c)c_{2,j,t+1} + s_{2,j,t+1} = w_{j,t+1}l_{2,j,t+1} - \tau_{2,j,t+1}(w_{j,t+1}l_{2,j,t+1} - s_{2,j,t+1}) + T_{2,j,t+1} \tag{12}$$

In the model of Armstrong et al. (2015), agents have a fixed tax rate and lump-sum payment based on their productivity. In contrast, in my model, taxable income determines the income tax rate ($\tau_{s,j,t}$) and lump-sum payment ($T_{s,j,t}$). As will be explained in Section 3, the average salary over both working periods of agent type j under EET is set to a calibration target. However, the agent can decide on the distribution of the income during the two working periods and the corresponding tax rate. Nevertheless, I assume the incomes to be higher than $\bar{y}_1$. For $s \in \{1,2\}$ the marginal tax rate $\tau_{s,j,t}$ is calculated as:

$$\tau_{s,j,t} = \begin{cases} \tau_1 & \text{if} \quad \bar{y}_1 < w_{j,t}l_{s,j,t} - s_{s,j,t} \leq \bar{y}_2 \\ \tau_2 & \text{if} \quad \bar{y}_2 < w_{j,t}l_{s,j,t} - s_{s,j,t} \leq \bar{y}_3 \\ \tau_3 & \text{if} \quad \bar{y}_3 < w_{j,t}l_{s,j,t} - s_{s,j,t} \end{cases} \tag{13}$$

$\bar{y}_1$, $\bar{y}_2$ and $\bar{y}_3$ are the income theresholds corresponding to the tax brackets. When the taxable salary exceeds a tax bracket, the individual faces an elevated tax rate. However, the agents pays a fixed income tax rae over his full income, while income in lower tax brackets are subject to a lower tax rate. $T_{s,j,t}$ is the lump sum, which accounts for lower tax rates in lower tax bands. Agents with a taxable salary above the threshold $\bar{y}_3$ don't benefit from a tax-free income of $\bar{y}_1$. For $s \in \{1,2\}$ the lump-sum is calculated as:

$$T_{s,j,t} = \begin{cases} \tau_{e1}\bar{y}_1 & \text{if} \quad \bar{y}_1 < w_{j,t}l_{s,j,t} - s_{s,j,t} \leq \bar{y}_2 \\ \tau_{e1}\bar{y}_1 + \tau_{e2}\bar{y}_2 & \text{if} \quad \bar{y}_2 < w_{j,t}l_{s,j,t} - s_{s,j,t} \leq \bar{y}_3 \\ \tau_{e2}\bar{y}_2 + \tau_{e3}\bar{y}_3 & \text{if} \quad \bar{y}_3 < w_{j,t}l_{s,j,t} - s_{s,j,t} \end{cases} \tag{14}$$

Substituting out savings, $s_{1,j,t}$ and $s_{2,j,t+1}$, using equation (6), gives the budget constraints:

$$(1+\tau_c)c_{1,j,t} = (1-\tau_{1,j,t})w_{j,t}l_{1,j,t} - (1-\tau_{1,j,t})\psi_{1,t}k_{2,j,t+1} + T_{1,j,t} \tag{15}$$

$$\begin{aligned}(1+\tau_c)c_{2,j,t+1} = {} & (1-\tau_{2,j,t})w_{j,t+1}l_{2,j,t+1} \\ & - (1-\tau_{2,j,t})\left(\psi_{2,t+1}k_{3,j,t+2} - k_{2,j,t+1}(1+r_{t+1}-\delta)\right) + T_{2,j,t+1}\end{aligned} \tag{16}$$

In retirement, agents consume all pension savings and state pension fund $\tilde{b}$. The budget constraint in retirement becomes:

$$(1+\tau_c)c_{3,j,t+2} = (1+r_{t+2}-\delta)k_{3,j,t+2}(1-\tau_{3,j,t+2}) + \tilde{b}(1-\tau_{3,j,t+2}) + T_{3,j,t+2} \tag{17}$$

Here $\tau_{3,j,t+2}$ is the marginal tax rate at retirement for agent type j at time $t+2$. Also here, the tax rate can vary depending on the taxable income upon retirement. Therefore:

$$\tau_{3,j,t+2} = \begin{cases} \tau_{ret1} & \text{if} \quad \bar{y}_1 < (1+r_{t+2}-\delta)k_{3,j,t+2} + \tilde{b} \leq \bar{y}_2 \\ \tau_{ret2} & \text{if} \quad \bar{y}_2 < (1+r_{t+2}-\delta)k_{3,j,t+2} + \tilde{b} \leq \bar{y}_3 \\ \tau_{ret3} & \text{if} \quad \bar{y}_3 < (1+r_{t+2}-\delta)k_{3,j,t+2} + \tilde{b} \end{cases} \tag{18}$$

$T_{3,j,t+2}$ is the lump sum which accounts for lower tax rates in lower tax bands for agent type j at retirement at time $t+2$.

$$T_{3,j,t+2} = \begin{cases} \tau^e_{ret1}\bar{y}_1 & \text{if} \quad \bar{y}_1 < (1+r_{t+2}-\delta)k_{3,j,t+2} + \tilde{b} \leq \bar{y}_2 \\ \tau^e_{ret1}\bar{y}_1 + \tau^e_{ret2}\bar{y}_2 & \text{if} \quad \bar{y}_2 < (1+r_{t+2}-\delta)k_{3,j,t+2} + \tilde{b} \leq \bar{y}_3 \\ \tau^e_{ret2}\bar{y}^*_2 + \tau^e_{ret3}\bar{y}_3 & \text{if} \quad \bar{y}_3 < (1+r_{t+2}-\delta)k_{3,j,t+2} + \tilde{b} \end{cases} \tag{19}$$

For an **employed** agent with mandatory pension savings, the two additional constraints are:

$$\begin{aligned} s_{1,j,t} &= \psi_{1,t}k_{2,j,t+1} \geq \phi(w_{j,t}l_{1,j,t} - f) \\ s_{2.j,t+1} &= \psi_{2,t+1}k_{3,j,t+2} - k_{2,j,t+1}(1+r_{t+1}-\delta) \geq \phi(w_{j,t+1}l_{2,j,t+1} - f) \end{aligned} \tag{20}$$

Here, $\phi(w_{j,t}l_{s,j,t} - f)$ for $s \in \{1,2\}$ is the mandatory savings calculated as a percentage (ϕ) of the pensionable base ($w_{j,t}l_{s,j,t} - f$), as explained in Section 2.1.

For a **self-employed** agent with voluntary retirement savings, the two additional constraints are:

$$s_{1,j,t} = \psi_{1,t} k_{2,j,t+1} \geq 0$$
$$s_{2.j,t+1} = \psi_{2,t+1} k_{3,j,t+2} - k_{2,j,t+1}(1 + r_{t+1} - \delta) \geq 0$$

This accounts for the fact that the pension funds are non-accessible before retirement.

Theorem 2.2. *Assuming pension capital division among survival peers in the case of death and Cobb-Douglas multiplicative, non-seperable preferences among agents. In a general overlapping generations model with an EET tax system, the optimal consumption and savings of a type j agent who anticipates his own future survival rate, but also that of his peers, can be determined by eight Lagrange multiplier equations. These Euler equations differ for employees participating in mandatory workplace pension schemes, contributing at least a percentage ϕ of their pensionable base, and self-employed, who only face the restriction that pension savings are non-accessible before retirement.*

The first two Lagrange multiplier equations describe the individual's optimal labour-leisure choice in the two working-aged periods $s = 1, 2$.

*For an **employed** agent with mandatory pension savings, the first two Lagrange multiplier equations are given by:*

$$\left[(c_{1,j,t})^{\gamma}(1 - l_{1,j,t})^{1-\gamma}\right]^{-\sigma} \tag{21}$$
$$\left[(1 - l_{1,j,t})^{1-\gamma}\gamma(c_{1,j,t})^{\gamma-1}\frac{(1 - \tau_{1,j,t})}{(1 + \tau_c)}w_{j,t} - (c_{1,j,t})^{\gamma}(1 - \gamma)(1 - l_{1,j,t})^{-\gamma}\right] + \lambda_{j,t}\phi w_{j,t} = 0$$

$$\beta\psi_{1,t}\left[(c_{2,j,t+1})^{\gamma}(1 - l_{2,j,t+1})^{1-\gamma}\right]^{-\sigma} \tag{22}$$
$$\left[(1 - l_{2,j,t+1})^{1-\gamma}\gamma(c_{2,j,t+1})^{\gamma-1}\frac{(1 - \tau_{2,j,t+1})}{(1 + \tau_c)}w_{j,t+1} - (c_{2,j,t+1})^{\gamma}(1 - \gamma)(1 - l_{2,j,t+1})^{-\gamma}\right] + \mu_{j,t+1}\phi w_{j,t+1} = 0$$

*For a **self-employed** agent who saves voluntarily for retirement, the first two Lagrange multiplier equations are given by:*

$$\frac{(1 - \tau_{1,j,t})}{(1 + \tau_c)}w_{j,t} = \frac{1 - \gamma}{\gamma}\frac{c_{1,j,t}}{1 - l_{1,j,t}} \tag{23}$$
$$\frac{(1 - \tau_{2,j,t+1})}{(1 + \tau_c)}w_{j,t+1} = \frac{1 - \gamma}{\gamma}\frac{c_{2,j,t+1}}{1 - l_{2,j,t+1}} \tag{24}$$

The third and fourth Lagrange multiplier equations determine the individual's optimal consumption-savings choice in the two working-aged periods $s = 1, 2$. They are equivalent for both employees and self-employed.

$$\begin{aligned} &-\frac{\psi_{1,t}(1 - \tau_{1,j,t})}{(1 + \tau_c)}(1 - l_{1,j,t})^{1-\gamma}\gamma(c_{1,j,t})^{\gamma-1}\left((c_{1,j,t})^{\gamma}(1 - l_{1,j,t})^{1-\gamma}\right)^{-\sigma} \\ &\quad +\beta\psi_{1,t}(1-l_{2,j,t+1})^{1-\gamma}\gamma(c_{2,j,t+1})^{\gamma-1}((c_{2,j,t+1})^{\gamma}(1-l_{2,j,t+1})^{1-\gamma})^{-\sigma}\frac{(1 - \tau_{2,j,t+1})}{(1 + \tau_c)}(1+r_{t+1}-\delta) \\ &\quad - \lambda_{j,t}\psi_{1,t} + \mu_{j,t+1}(1 + r_{t+1} - \delta) = 0 \end{aligned} \tag{25}$$

$$- \psi_{1,t}\beta\frac{\psi_{2,t+1}(1-\tau_{2,j,t+1})}{(1+\tau_c)}(1-l_{2,j,t+1})^{1-\gamma}\gamma(c_{2,j,t+1})^{\gamma-1}\left((c_{2,j,t+1})^{\gamma}(1-l_{2,j,t+1})^{1-\gamma}\right)^{-\sigma}$$
$$+ \beta^2\psi_{1,t}\psi_{2,t+1}\gamma(c_{3,j,t+2})^{\gamma(1-\sigma)-1}(1+r_{t+2}-\delta)\frac{(1-\tau_{3,j,t+2})}{(1+\tau_c)}$$
$$- \mu_{j,t+1}\psi_{2,t+1} = 0 \quad (26)$$

For an ***employed*** *agent, the last four Lagrange multiplier equations are:*

$$\psi_{1,t}k_{2,j,t+1} - \phi(w_{j,t}l_{1,j,t} - f) - s_j^2 = 0$$
$$\psi_{2,t+1}k_{3,j,t+2} - k_{2,j,t+1}(1+r_{t+1}-\delta) - \phi(w_{j,t+1}l_{2,j,t+1} - f) - t_j^2 = 0$$
$$2\lambda_{j,t}s = 0$$
$$2\mu_{j,t+1}t = 0$$

For a ***self-employed*** *agent, the last four Lagrange multiplier equations are:*

$$\psi_{1,t}k_{2,j,t+1} - s_j^2 = 0$$
$$\psi_{2,t+1}k_{3,j,t+2} - k_{2,j,t+1}(1+r_{t+1}-\delta) - t_j^2 = 0$$
$$2\lambda_{j,t}s_j = 0$$
$$2\mu_{j,t+1}t_j = 0$$

Note. All mathematical abbreviations can be found in Table 1.

Proof. The proof of Theorem 2.2 can be found in Appendix A.3. □

2.3.2 Tax-Exempt-Exempt (TEE) pension tax system

I continue outlining the TEE tax system, were pension contributions are taxed instead of the pension benefits. The following budget constraints of a working agent show that savings are made from disposable income.

$$(1+\tau_c)c_{1,j,t} + s_{1,j,t} = w_{j,t}l_{1,j,t}(1-\tau_{1,j,t}) + T_{1,j,t} \quad (27)$$
$$(1+\tau_c)c_{2,j,t+1} + s_{2,j,t+1} = w_{j,t+1}l_{2,j,t+1}(1-\tau_{2,j,t+1}) + T_{2,j,t+1} \quad (28)$$

The marginal tax rate $\tau_{s,j,t}$ at time t can again vary during working life for agent type j and is dependent on his taxable income. Hence, for $s \in \{1,2\}$:

$$\tau_{s,j,t} = \begin{cases} \tau_1 & \text{if} \quad \bar{y}_1 < w_{j,t}l_{s,j,t} \leq \bar{y}_2 \\ \tau_2 & \text{if} \quad \bar{y}_2 < w_{j,t}l_{s,j,t} \leq \bar{y}_3 \\ \tau_3 & \text{if} \quad \bar{y}_3 < w_{j,t}l_{s,j,t} \end{cases} \quad (29)$$

And the lump-sum payments for $s \in \{1,2\}$ are calculated as:

$$T_{s,j,t} = \begin{cases} \tau_{e1}\bar{y}_1 & \text{if} \quad \bar{y}_1 < w_{j,t}l_{s,j,t} \leq \bar{y}_2 \\ \tau_{e1}\bar{y}_1 + \tau_{e2}\bar{y}_2 & \text{if} \quad \bar{y}_2 < w_{j,t}l_{s,j,t} \leq \bar{y}_3 \\ \tau_{e2}\bar{y}_2 + \tau_{e3}\bar{y}_3 & \text{if} \quad \bar{y}_3 < w_{j,t}l_{s,j,t} \end{cases} \quad (30)$$

Again, $\bar{y}_1$, $\bar{y}_2$ and $\bar{y}_3$ are the income thresholds corresponding to the tax brackets. When the taxable salary exceeds a tax bracket, the individual faces an elevated tax rate. Like under EET, the lump-sum payment $T_{s,j,t}$ accounts for lower tax rates in lower tax brackets. Moreover, agents with a taxable salary above the threshold $\bar{y}_3$ don't benefit from a tax-free income of $\bar{y}_1$.

As mentioned in Section 2.3, the government tops up the agent's pension savings with a certain percentage. This pension saving subsidy, τ_b, is introduced to stimulate pension savings under TEE. Then, pension capital accumulates over working life as:

$$k_{s+1,j,t+1} = \frac{1}{\psi_{s,t}}(k_{s,j,t}(1 + r_t + \delta) + s_{s,j,t}(1 + \tau_b)) \tag{31}$$

Substituting out the savings in the working-aged budget constraints under TEE with pension savings top-up rate τ_b results in:

$$(1 + \tau_c)c_{1,j,t} + \frac{\psi_{1,t}k_{2,j,t+1}}{1 + \tau_b} = w_{j,t}l_{1,j,t}(1 - \tau_{1,j,t}) + T_{1,j,t} \tag{32}$$

$$(1 + \tau_c)c_{2,j,t+1} + \frac{\psi_{2,t+1}k_{3,j,t+2} - k_{2,j,t+1}(1 + r_{t+1} - \delta)}{1 + \tau_b} = w_{j,t+1}l_{2,j,t+1}(1 - \tau_{2,j,t+1}) + T_{2,j,t+1} \tag{33}$$

In retirement, the agents consume their accumulated pension capital and state pension fund. In contrast to under EET, the (asccumulated) pension benefits are not taxed. However, as like under EET, the state pension is taxed as it lies above the tax-free threshold. The state pension lies within the first tax bracket. Therefore, $\tau_{3,j,t} = \tau_{ret1}$ for all $j \in \{1, 2, 3\}$. The retirement budget constraint becomes:

$$(1 + \tau_c)c_{3,j,t+2} = (1 + r_{t+2} - \delta)k_{3,j,t+2} + \tilde{b}(1 - \tau_{ret1}) + T^r_{j,t+2} \tag{34}$$

Moreover, similar to under EET, agents with a taxable salary above the income threshold $\bar{y}_3$ do not benefit from a tax-free income of $\bar{y}_1$. Additionally, $\bar{y}_1 > \tilde{b}$. Therefore:

$$T_{3,j,t+2} = \tau^e_{ret1}\bar{y}_1 \qquad \text{if} \quad (1 + r_{t+2} - \delta)k_{3,j,t+2} + \tilde{b} \leq \bar{y}_3 \tag{35}$$

For an **employed** agent with mandatory pension savings, the two additional constraints are:

$$s_{1,j,t}(1 + \tau_b) = \psi_{1,t}k_{2,j,t+1} \geq \phi(w_{j,t}l_{1,j,t} - f)(1 - \tau_{retj}) \tag{36}$$
$$s_{2.j,t+1}(1 + \tau_b) = \psi_{2,t+1}k_{3,j,t+2} - k_{2,j,t+1}(1 + r_{t+1} - \delta) \geq \phi(w_{j,t+1}l_{2,j,t+1} - f)(1 - \tau_{retj})$$

Remark. The pension savings have to be, *in addition to the pension subsidy*, greater than, or equal to, the mandatory contributions of the workplace pension. I multiply the mandatory pension savings seen under EET ($\phi(w_{j,t+1}l_{2,j,t+1} - f)$), with $(1 - \tau_{retj})$. With this adjustment, I take into account that pension savings under TEE are not tax exempted, while the pension benefits under EET are. Here, τ_{retj} is dependent on the agents' income and equal to the highest tax rate the same agent would have to pay under EET at retirement. With this adjustment, I do not take into account the lower tax rates in the lower tax brackets. [5]

[5] Taking tax brackets into account, agents would save more. However, the results in Section 5, show that also without this adjustment, employees have a high replacement rate.

I clarify the fairness of this adjustment with an example. Let's assume, under the EET system, an employee saves a total of €10,000 for retirement in a particular year. These savings are tax-exempted, while the accrued pension benefits received at retirement are not. When he retires 20 years later, and has to pay 30% income tax on his pension benefits, then, assuming an equal annual interest rate, his total net pension benefits amount to $10,000 \times (1+r)^{20} \times 0.7$. Under TEE, with the same interest rate, the amount of pension contribution (x) that is necessary to receive the same net pension can be calculated via the following equation:

$$10,000 \times (1+r)^{20} \times 0.7 = x \times (1+r)^{20}$$

$$\Longleftrightarrow$$

$$x = \frac{10,000 \times (1+r)^{20} \times 0.7}{(1+r)^{20}} = 10,000 \times 0.7$$

For a **self-employed** agent with voluntary retirement savings, the two additional constraints are:

$$s_{1,j,t}(1+\tau_b) = \psi_{1,t} k_{2,j,t+1} \geq 0$$
$$s_{2.j,t+1}(1+\tau_b) = \psi_{2,t+1} k_{3,j,t+2} - k_{2,j,t+1}(1+r_{t+1}-\delta) \geq 0$$

Theorem 2.3. *Assuming pension capital division among survival peers in the case of death and Cobb-Douglas multiplicative, non-seperable preferences among agents. In a general overlapping generations model with a TEE tax system, the optimal consumption and savings of a type j agent who anticipates his own future survival rate, but also that of his peers, can be determined by eight Lagrange multiplier equations. These Euler equations are different for employees with mandatory savings, contributing at least a percentage ϕ of their pensionable base, and self-employed, who only have the restriction that pension savings are non-accessible before retirement.*

The first two Lagrange multiplier equations describe the individual's optimal labour-leisure choice in the two working-aged periods $s = 1, 2$.

*For an **employed** agent with mandatory pension savings, the first two Lagrange multiplier equations are given by:*

$$\left[(c_{1,j,t})^{\gamma}(1-l_{1,j,t})^{1-\gamma}\right]^{-\sigma} \left[(1-l_{1,j,t})^{1-\gamma}\gamma(c_{1,j,t})^{\gamma-1}\frac{(1-\tau_{1,j,t})}{(1+\tau_c)}w_{j,t} - (c_{1,j,t})^{\gamma}(1-\gamma)(1-l_{1,j,t})^{-\gamma}\right] + \lambda_{j,t}\phi w_{j,t} = 0 \tag{37}$$

$$\beta\psi_{1,t}\left[(c_{2,j,t+1})^{\gamma}(1-l_{2,j,t+1})^{1-\gamma}\right]^{-\sigma} \left[(1-l_{2,j,t+1})^{1-\gamma}\gamma(c_{2,j,t+1})^{\gamma-1}\frac{(1-\tau_{2,j,t+1})}{(1+\tau_c)}w_{j,t+1} - (c_{2,j,t+1})^{\gamma}(1-\gamma)(1-l_{2,j,t+1})^{-\gamma}\right] + \mu_{j,t+1}\phi w_{j,t+1} = 0 \tag{38}$$

*For a **self-employed** agent who saves voluntarily for retirement, the first two Lagrange multiplier equations are given by:*

$$\frac{(1-\tau_{1,j,t})}{(1+\tau_c)}w_{j,t} = \frac{1-\gamma}{\gamma}\frac{c_{1,j,t}}{1-l_{1,j,t}} \tag{39}$$

$$\frac{(1-\tau_{2,j,t+1})}{(1+\tau_c)}w_{j,t+1} = \frac{1-\gamma}{\gamma}\frac{c_{2,j,t+1}}{1-l_{2,j,t+1}} \tag{40}$$

The third and fourth Lagrange multiplier equations determine the individual's optimal consumption-savings choice in the two working-aged periods $s = 1, 2$. They are equivalent for both the employee and self-employed.

$$\begin{aligned} &-\left((c_{1,j,t})^{\gamma}(1-l_{1,j,t})^{1-\gamma}\right)^{-\sigma}\gamma(c_{1,j,t})^{\gamma-1}(1-l_{1,j,t})^{1-\gamma}\frac{\psi_{1,t}}{(1+\tau_b)(1+\tau_c)} \\ &\quad +\beta\psi_{1,t}\left((c_{2,j,t+1})^{\gamma}(1-l_{2,j,t+1})^{1-\gamma}\right)^{-\sigma}\gamma(c_{2,j,t+1})^{\gamma-1}(1-l_{2,j,t+1})^{1-\gamma} \\ &\qquad \frac{1}{(1+\tau_b)(1+\tau_c)}(1+r_{t+1}-\delta) - \lambda_{j,t}\psi_{1,t} + \mu_{j,t+1}(1+r_{t+1}-\delta) = 0 \end{aligned} \tag{41}$$

$$\begin{aligned} &-\beta\psi_{1,t}\left((c_{2,j,t+1})^{\gamma}(1-l_{2,j,t+1})^{1-\gamma}\right)^{-\sigma}\gamma(c_{2,j,t+1})^{\gamma-1}(1-l_{2,j,t+1})^{1-\gamma}\frac{\psi_{2,t+1}}{(1+\tau_b)(1+\tau_c)} \\ &\quad +\beta^2\psi_{1,t}\psi_{2,t+1}\gamma(c_{3,j,t+2})^{\gamma(1-\sigma)-1}\frac{1}{(1+\tau_c)}(1+r_{t+2}-\delta) - \mu_{j,t+1}\psi_{2,t+1} = 0 \end{aligned} \tag{42}$$

*For an **employed** agent, the last four Lagrange multiplier equations are:*

$$\begin{aligned} &\psi_{1,t}k_{2,j,t+1} - \phi(w_{j,t}l_{1,j,t} - f)(1-\tau_{retj}) - s_j^2 = 0 \\ &\psi_{2,t+1}k_{3,j,t+2} - k_{2,j,t+1}(1+r_{t+1}-\delta) - \phi(w_{j,t+1}l_{2,j,t+1} - f)(1-\tau_{retj}) - t_j^2 = 0 \\ &2\lambda_{j,t}s = 0 \\ &2\mu_{j,t+1}t = 0 \end{aligned}$$

*For a **self-employed**, the last four Lagrange multiplier equations are:*

$$\begin{aligned} &\psi_{1,t}k_{2,j,t+1} - s_j^2 = 0 \\ &\psi_{2,t+1}k_{3,j,t+2} - k_{2,j,t+1}(1+r_{t+1}-\delta) - t_j^2 = 0 \\ &2\lambda_{j,t}s = 0 \\ &2\mu_{j,t+1}t = 0 \end{aligned}$$

Note. All mathematical abbreviations can be found in Table 1.

Proof. The proof of Theorem 2.3 can be found in Appendix A.3. □

2.4 Market clearing

There are three markets in the model: goods, labour and capital. It is assumed that the market for labour and capital instantly clears itself. The size of the age groups and the different decisions of the employees and self-employed are taken into account. The corresponding market clearing functions are:

$$L_t = \sum_{s=1,2} \sum_{j=1,2,3} pop_{s,t}\pi_j \rho_j (\Phi l^{e}_{s,j,t} + (1-\Phi) l^{s}_{s,j,t}) \tag{43}$$

$$K_t = \sum_{s=2,3} \sum_{j=1,2,3} pop_{s,t}\pi_j (\Phi k^{e}_{s,j,t} + (1-\Phi) k^{s}_{s,j,t}) \tag{44}$$

Here, L_t is the effective labour which is the labour supply weighted by agents' productivities. K_t is the total amount of accumulated pension capital in the model at time t. The superscripts e and s denote the decisions of the employed and self-employed agents, respectively. Moreover, recall that labour at retirement, $l_{3,j,t}$, and initial pension saving, $k_{1,j,t}$, are zero.

By Walras' law, the market clearing condition for the good market is redundant. The market clearing condition for the good market determines the output of the economy. Armstrong et al. (2015) calculate the output as $Y_t = C_t + S_t + B_t + G_t$. Here, C_t represents total household consumption, S_t denotes the aggregate savings, B_t is the aggregate pension payment, and G_t denotes the government spending. The subsidy is not included as $T_t = B_t + G_t$ represents the *net* tax revenue, reflecting the tax revenue minus any subsidies (as will be outlined in Section 2.6).[6] Moreover, the state pension expenditures (B_t) should not be included in the output calculation as it is already included in the consumption (C_t). Lastly, Armstrong et al. (2015) do not take into account that the available resources are not only the output of the economy, but also the non-depreciated capital seen at time t. All in all, the presented economic output of Armstrong et al. (2015), resulting from their goods clearance function, is not equal to the economy's output resulting from the Cobb-Douglas production function. In the Discussion (Section 8), I argue how this inaccuracy could have influenced Armstrong et al. (2015) conclusion.

Adapting the market clearing function of Groth (2016) to incorporate government expenditures results in the following good market clearing condition:

$$Y_t = C_t + G_t + K_{t+1} - (1-\delta)K_t \tag{45}$$

Under the *steady-state*, the output of the economy can be calculated as: $Y = C + G + \delta K$. Moreover, $K_{t+1} - (1-\delta)K_t$ is the aggregate *net* investment (Groth, 2016) (not to be confused with investment I_t)). Calculating the aggregate net investment, the interest rate and the accumulation of capital as in equation (6) (the redivision after dead) is taken into account.

Calculating the output this way ensures both the inclusion of subsidy and the non-depreciated capital in the calculation of output. The economy's output shown in Section

[6] To be precise, Armstrong et al. (2015) calculate the output as $Y_t = C_t + I_t + B_t + G_t$. However, Armstrong et al. (2015) calculate I_t similarly to how I calculate S_t.

5, calculated using this good market clearing condition, is checked to be equal to the Cobb-Douglas production function.

Section 2.6 provides further elaboration on the government and its expenditures. C_t and B_t are calculated as follows:

$$C_t = \sum_{s=1,2,3} \sum_{j=1,2,3} pop_{s,t}\pi_j(\Phi c^{e}_{s,j,t} + (1-\Phi)c^{s}_{s,j,t}) \tag{46}$$

$$B_t = pop_{3,t}\tilde{b} \tag{47}$$

The aggregate savings are calculated as:

$$S_t = \sum_{s=1,2} \sum_{j=1,2,3} pop_{s,t}\pi_j(\Phi s^{e}_{s,j,t} + (1-\Phi)s^{s}_{s,j,t}) \tag{48}$$

The aggregate investment, I_t, at time t is closely related to the aggregate savings, S_t, at time t. Under EET, the aggregate savings are equal to the aggregate savings. Under TEE, the aggregate investment is calculated as:

$$I_t = (1+\tau_b)S_t \tag{49}$$

2.5 Firms

The model has identical, perfectly competitive firms. The firm maximises the Cobb Douglas production function $Y = AK^{\alpha}L^{1-\alpha}$ minus the costs on labour and capital: firms employ labour at the real wage rate w_t and capital at the real interest rate r_t.[7] I assume that firms pay no taxes. The firm maximizes its real profits at time t:

$$\max_{L_t,K_t} \quad A_t(L_t)^{1-\alpha}(K_t)^{\alpha} - r_tK_t - w_tL_t \tag{50}$$

From this equation, it is evident that firms are assumed to utilize all effective labour capacity at time t.

The first order condition w.r.t. L_t gives:

$$(1-\alpha)A_tL_t^{-\alpha}K_t^{\alpha} - w_t = 0$$

The first order condition w.r.t. K_t gives:

$$\alpha A_t(L_t)^{1-\alpha}K_t^{\alpha-1} - r_t = 0$$

Rewriting gives the real factor prices of wage and capital.

$$r_t = \alpha A_t(\frac{K_t}{L_t})^{\alpha-1} \tag{51}$$

$$w_t = (1-\alpha)A_t(\frac{K_t}{L_t})^{\alpha} \tag{52}$$

Both the wage and interest rates are expressed in real terms.

[7] $\alpha + (1-\alpha) = 1$, therefore the Cob Douglas production function has a constant returns to scale: if we increase L and K with the same factor, the output increases with the same factor.

2.6 Government

I assume that the government runs a balanced budget. Therefore, government expenditures alter such that income and expenses, both resulting from the model, are in balance again. Therefore, the following government constraint does not show up in the optimisation problem.

Under EET, tax revenues are raised on labour income net of pension savings, retirement income and consumption tax and are used for government expenditures (G_t) and the state pension (B_t). I adjust the government constraint used by Armstrong et al. (2015) under EET as Armstrong et al. (2015) taxes $k^s_{3,j,t}$ at retirement. However, I assume that the amount that the agent receives at retirement, the accumulated pension capital ($k^s_{3,j,t}(1+r_t-\delta)$), is taxed. This aligns with the budget constraint of an agent at retirement under EET (also seen in the model of Armstrong et al. (2015)):

$$(1+\tau_c)c_{3,j,t+2} = (1+r_{t+2}-\delta)k_{3,j,t+2}(1-\tau_{3,j,t+2}) + \tilde{b}(1-\tau_{3,j,t+2}) + T_{3,j,t+2} \tag{53}$$

This adjustment is needed for the accuracy of the results. Moreover, the size of the age groups and the different decisions of the employed and self-employed are taken in to account in the calculation. The government constraint becomes:

$$\begin{aligned} G_t + B_t = \Phi &\sum_{s=1,2}\sum_{j=1,2,3} pop_{s,t}\pi_j \left(\tau^e_{s,j,t}(w_{j,t}l^e_{s,j,t} - s^e_{s,j,t}) - T^e_{s,j,t}\right) \\ &+(1-\Phi)\sum_{s=1,2}\sum_{j=1,2,3} pop_{s,t}\pi_j \left(\tau^s_{s,j,t}(w_{j,t}l^s_{s,j,t} - s^s_{s,j,t}) - T^s_{s,j,t}\right) \\ &+\Phi\sum_{j=1,2,3} pop_{3,t}\pi_j \left(\tau^e_{3,j,t}(k^e_{3,j,t}(1+r_t-\delta)+\tilde{b}) - T^e_{3,j,t}\right) \\ &+(1-\Phi)\sum_{j=1,2,3} pop_{3,t}\pi_j \left(\tau^s_{3,j,t}(k^s_{3,j,t}(1+r_t-\delta)+\tilde{b}) - T^s_{3,j,t}\right) + \tau_c C_t \end{aligned} \tag{54}$$

Here, the state expenditures B_t is equal to $pop_{3,t}\tilde{b}$. The first two terms on the right side denote the tax revenue on working employees and self-employed respectively. The third and fourth terms on the right side are the tax revenue on retired employees and self-employed. Lastly, $\tau_c C_t$ is the revenue from consumption taxes.

Under TEE, government expenditures and the state pension are paid from labour tax revenue, taxes on state pensions and consumption tax revenue, net of pension savings subsidy, $\tau_b S_t$. The government's budget constraint is:

$$\begin{aligned} G_t + B_t =&\Phi\sum_{s=1,2}\sum_{j=1,2,3} pop_{s,t}\pi_j(\tau^e_{s,j,t}w_{j,t}l^e_{s,j,t} - T^e_{s,j,t}) \\ &+(1-\Phi)\sum_{s=1,2}\sum_{j=1,2,3} pop_{s,t}\pi_j(\tau^s_{s,j,t}w_{j,t}l^s_{s,j,t} - T^s_{s,j,t}) \\ &+\Phi\sum_{j=1,2,3} pop_{3,t}\pi_j\left(\tau_{ret1}\tilde{b} - T^e_{3,j,t}\right) + (1-\Phi)\sum_{j=1,2,3} pop_{3,t}\pi_j\left(\tau_{ret1}\tilde{b} - T^s_{3,j,t}\right) - \tau_b S_t + \tau_c C_t \end{aligned} \tag{55}$$

Under both the EET and TEE tax system the **net** tax revenue is calculated as:

$$T = G + B = G + pop_{3,t}\tilde{b} \tag{56}$$

And is therefore equal the total tax revenue minus any subsidies.

2.7 Steady-state and perfect-foresight simulations in Dynare

In this Section, I explain the so-called steady state and the perfect foresight simulation and introduce the algorithms behind the calculations.

In the previous Section, I defined a general equilibrium overlapping generations model. To generate a general equilibrium, using all model equations, endogenous variables and parameters, I use *Dynare*. Dynare is a software platform designed to manage a diverse range of economic models, with a specific focus on dynamic stochastic general equilibrium (DSGE) and overlapping generations (OLG) models (Dynare, 2024).

In Section 5.1, I show the so-called *steady-state simulation results* with the parameters calibrated to represent 2019, which is in the model time $t = 0$. The steady state is determined by all agents maximising their lifetime utility when entering the model by adjusting their savings and labour. Agents take the probability of survival, as seen at $t = 0$, into account, and do not have the opportunity to revise their decision after the realisation of the last period. In the steady state equilibrium, all endogenous parameters remain constant. Dynare computes the steady state using a nonlinear Newton-type solver, an iterative procedure that takes an initial guess as input (Dynare Team, 2023). One iteration involves solving the more straightforward linear equation obtained by the first-order approximation of the mapping around the current iterate (Izmailov and Solodov, 2015).[8]

In Section 5.2, I present for different demographic scenarios *perfect foresight simulation results* of the economy under both tax systems over time, taking into account projected demographics. The demographics change over time, but deterministically, with changes determined in Section 3. In each period, Dynare calculates a new equilibrium in anticipation of, and in reaction to, the shock. Agents have perfect foresight of how the survival rate and the population change, which they take into account in their optimisation problem (no need for adjustment). The simulation results at $t = 0$ are equal to the steady-state results (input), after which the algorithm computes the new equilibriums at time $t = 1$, $t = 2$ and $t = 3$. [9]

In only a few instances, *Dynare* has difficulty finding an optimum due to abrupt changes in the derivatives caused by discontinuities in the tax brackets. Therefore, in these particular cases, I decide to set the salary of the particular agent ($l_{s,j,t}w_{j,t}$) equal to the boundary of the tax brackets by adjusting the time spent working ($l_{s,j,t}$). I specify in the results when this occurs. All calculations can be found in Appendix A.4. The notation of capital accumulation differs in the code, as one preliminary of Dynare is that all variables known at time t must be dated t-1. Moreover, lagged variables are denoted by $y(-1)$.

[8] I use the default nonlinear Newton solver in dynare, which divides the model into recursive blocks and sequentially solves each block using a trust-region solver with autoscaling (Dynare Team, 2023). The iteration ceases when the residuals of all equations are smaller than the default value, $eps^{1/3}$ (Dynare Team, 2023). Here, eps equals 2^{-52} (MathWorks, 2024).

[9] I use the default Newton-type algorithm. The algorithm ceases when it is impossible to improve the function by more than the default value $1e - 5$ or when the solver attempts to take a step smaller than the default value $1e - 5$.

3 Calibration

This section presents a fully calibrated model that resembles the economy of the Netherlands in 2019. Due to Covid-19 measures in 2020 and the subsequent economic pressure, I believe the economy during the pandemic is not representative of the current economy. With the last restrictions ending in 2022 and the 2023 annual report not published yet, I use 2019 as the baseline year. I calibrate the parameters to realised values in 2019 or I set parameters such that particular target goals under EET are reached. The remaining parameters are set equal to commonly accepted values used in related macroeconomic research. As currently the EET tax system is implemented in the Netherlands, I can only calibrate the OLG model with the EET tax system to particular goals. However, I assume that the calibrated parameters are intrinsic values under EET and TEE. By keeping the input the same, I can observe how the output changes due to changing policy while the agents share the same values.

Moreover, to measure the economic consequences of demographic changes on the different pension tax models, it is essential to make realistic projections on all parameters that drive demography. I calculate the expectations for future survival rates via the well-known Lee-Carter method. Expectations on the future fertility rate and migration are set in line with the expectations of CBS.

The following Table 2 presents all exogenous parameters along with the corresponding sections where I discuss them.

Table 2: Exogenous parameter values to be calibrated

Section	**Parameter**	**Description**
section 3.1	τ_c	consumption tax rate
	$(\tau_1, \tau_2, \tau_3, {\tau_1}^e, {\tau_2}^e, {\tau_3}^e)$	(excess) tax income rates at working age
	$(\tau_{ret1}, \tau_{ret2}, \tau_{ret3}, \tau^e_{ret1}, \tau^e_{ret2}, \tau^e_{ret3})$	(excess) tax income rates at retirement
	(π_1, π_2, π_3)	share agents $j = 1, 2, 3$ in population
	$(\bar{y}_1, \bar{y}_2, \bar{y}_3)$	income thresholds tax brackets
section 3.2	$pop_{s,0}$	initial size age groups, for s $\in \{1, 2, 3\}$
	Φ	ratio employed
	$\psi_{s,t}$	for every $s \in \{1, 2, 3\}$ and $t \in \{0, 1, 2, 3\}$
	fr_t	for every $t \in \{0, 1, 2, 3\}$
	mr_t	for every $t \in \{0, 1, 2, 3\}$
section 3.3	γ	weight on leisure
	σ	consumption smoothing
	β	discount factor
	(ρ_1, ρ_2, ρ_3)	productivity agents $j = 1, 2, 3$
section 3.4	α	capital share of income
	A	technology factor
	δ	depreciation rate
section 3.5	ϕ	contribution rate workplace pension
	$\tilde{b}$	state pension (AOW)
	f	franchise

For parameters like $\psi_{s,t}$, fr_t and mr_t, it is especially important to keep in mind that a period represents 20 years. When $t = 0$ represents the year 2019, the difference between $\psi_{s,0}$ and $\psi_{s,1}$ is the difference in the survival rate for a particular age group s to reach age group $s + 1$ in the year 2019 and year 2039. All parameter values with $t \in \{1, 2, 3\}$, are thus expectations in the future. As $t = 0$ represents 2019, $t = 1$, $t = 2$, and $t = 3$ can be seen as the years 2039, 2059 and 2079, respectively.

3.1 Tax scheme

In this Section, I explain the progressive tax regime used in the model. As described in Section 2.2, I distinguish between three different tax bracket corresponding to the basic, higher and additional income tax rate.

In 2020, the Netherlands introduced two tax brackets for non-retirees and three tax brackets for retirees, which were three and four, respectively, in 2019. I have chosen to use the tax brackets established in the year 2020, considering the three distinct incomes of the three agents, all matching the average income of a tax bracket.[10] All income tax rates are retrieved from a published paper by the Dutch Ministry of Finance (Ministerie van Financiën, 2020). Moreover, I take into account that for many agents, the first part of the income is tax-free. In 2019, this tax-free personal allowance was approximately €7,100 (Financieel infonu, 2021). However, for agents with income crossing €68,507 ($\bar{y}_1$), the tax-free income is neglectable. Therefore, with the use of the lump-sum payments described in Section 2, in the OLG model, only agents with earnings lower than €68,507 benefit from the tax-free income.

Table 3: Income tax rates and personal allowance

	Income	**Income Tax**	
		Working class	Pensioners
Tax-free	< €7,100	0%	0%
Basic rate	€7,100 to €34,712	37.35%	19.45 %
Higher rate	€34,712 to €68,507	37.35%	37.35 %
Additional rate	> €68,507	49.50%	49.50 %

As can be seen, the income tax rate for income in the first tax bracket is higher for the working population than for pensioners. This is because the working population pays for the state pension of current retirees arranged via a pay-as-you-go system.

The productivity of agent type j is set such that the average wage he earns during working life is consistent with the average salary seen in tax bracket j, which I explain in detail in Section 3.3. In the following Table 4, the characteristics of agent type j are displayed. The average *gross* incomes of the employees and self-employed with gross incomes falling in the tax brackets are shown.[11] Furthermore, the share of agent type j in the population is shown.

[10]Neglecting the influence of one years' inflation on the income thresholds of the income brackets.

[11]The average income is calculated as the weighted average of the *gross* incomes seen in the tax bracket, as reported by CBS (CBS, 2019b). In the calculation, I included the data of both employees and self-employed and excluded gross income to €10,000 per year.

Table 4: Income and share tax bands Netherlands 2019

Agent type j	**Tax bracket**	**Average gross income**	$w_{j,t}$ **in model**	**Share agent j** (π_j)
1	Basic rate	€24,552	$\rho_1 * w_t$	32.05%
2	Higher rate	€50,990	$\rho_2 * w_t$	37.08%
3	Additional rate	€90,140	$\rho_3 * w_t$	30.88%

In the model, the agents pay the marginal tax rate τ_l^j over their full taxable income. To account for lower income tax rate for income falling into lower tax brackets, this is compensated with a certain amount of lump-sum $T_{s,j,t}$. All details are outlined in Table 5.

Table 5: Marginal tax rate and lump sum

Tax bracket	**Marginal tax rate**		**Lump-sum**	
	Working class (τ_j)	Retired (τ_{retj})	Working class ($T_{s,j,t}$)	Retired ($T_{3,j,t}$)
Basic rate	37.35%	19.45 %	$\tau_1^e \bar{y}_1$	$\tau_{ret1}^e \bar{y}_1$
Higher rate	37.35%	37.35%	$\tau_1^e \bar{y}_1 + \tau_2^e \bar{y}_2$	$\tau_{ret1}^e \bar{y}_1 + \tau_{ret2}^e \bar{y}_2$
Additional rate	49.50%	49.50%	$\tau_2^e \bar{y}_2 + \tau_3^e \bar{y}_3$	$\tau_{ret2}^e \bar{y}_2 + \tau_{ret3}^e \bar{y}_3$

In the model, one euro represents one hundred thousand euros. Therefore, $\bar{y}_1$ is set to 0.07100. Likewise $\bar{y}_2 = 0.34712$ and $\bar{y}_3 = 0.68507$. Moreover, $\tau_1^e = 0.3735$, $\tau_2^e = 0$ and $\tau_3^e = 0.1215$, which are the differences between the different tax rates at working age. Similarly, at retirement, the differences between the tax rates are $\tau_{ret1}^e = 0.1945$, $\tau_{ret2}^e = 0.1790$ and $\tau_{ret3}^e = 0.1215$. As can be seen from the lump-sum $T_{3,j,t}$, for agents with earnings falling into the additional rate tax bracket, the tax-free personal allowance of €7,100 is phased out.

The consumption tax rate, τ_c is set equal to 0.21 as the general VAT rate (BTW in Dutch) of 21% applies to all products and services that are not exempt and do not fall under the 9 % or 0 % rate (Belastingdienst, 2024).

As a last remark, in the model, firms make no profit and are not taxed.

3.2 Demographic projection

As introduced in Section 2.2, the size of age group s at time t is given by:

$$pop_{s,t} = \begin{cases} pop_{1,t-1} fr_{t-1} + mr_t & \text{for } s = 1 \\ pop_{s-1,t-1} \psi_{s-1,t-1} & \text{for } s \in \{2, 3\} \\ 0 & \text{for } s > 3 \end{cases} \tag{57}$$

Each age group's size depends on the development of fertility rates, survival rates, and migration over time. First, I explain my approach to generate realistic projections of survival rates 60 years in the future in Section 3.2.2. I elaborate on the projected fertility and migration rates in Sections 3.2.3 and 3.2.2, respectively. In Section 3.2.1, I determine the initial population at $t = 0$ (representing 2019), its distribution among the age groups and the ratio of self-employed. Ultimately, these variables determine the projected population in the future.

3.2.1 Initial population

I do not include children in the model. In 2019, 13.486 million people were aged 20 or older, as retrieved from the population pyramid of the Dutch population in 2019 (CBS, 2023). I normalise in the model the total adult population at $t = 0$ to 1 (CBS, 2023).

In the steady state all endogenous variables remain constant and therefore, lags are irrelevant. From equation (1) the initial population therefore has to meet the following conditions: $pop_{2,0} = \psi_{1,0} pop_{1,0}$ and $pop_{3,0} = \psi_{2,0}\psi_{1,0} pop_{1,0}$. As shown in Section 3.2.2, the survival rates at time $t = 0$ are $\psi_{1,0} = 0.98045$ and $\psi_{2,0} = 0.89355$. Given the survival rates at time $t = 0$, equations (3.2.1) and (3.2.1), normalising the initial population to one gives the following distribution among the age groups:

$$(pop_{1,0}, pop_{2,0}, pop_{3,0}) = (0.35007, 0.34323, 0.30669) \tag{58}$$

Remark. The initial population must meet these criteria. Otherwise, the aggregate pension capital is wrongfully determined via equation (6) in the steady state.

With published data on income classes and person characteristics, I can determine Φ, the ratio of employees compared to self-employed. Looking at the data from 2019, 80.5% of the working population is an employee, and 19.5% is self-employed(CBS, 2019b). Therefore, $\Phi = 0.805$.

3.2.2 Survival rate projections

For forecasting future survival rates, I use the Lee-Carter method. The Lee-Carter method, developed by Ronald D. Lee and Lawrance R. Carter (1992), is a widely accepted probabilistic approach to forecast mortality, appreciated for its simplicity and straightforward interpretation of model parameters. The model assumes a constant age component and linear time component for forecasting (Rabbi and Mazzuco, 2021). I apply the Lee-Carter (LC) method for predicting future mortality rates, after which I calculate the survival rate as one minus the mortality rate. The LC method involves several steps to estimate the LC model and fit a time-series model to the time index. A comprehensive explanation of all calculations is provided by MathWorks, Incl. (The MathWorks, Inc., 2024). In this Section, I elaborate on the decisions I make to estimate the future survival rates of the agents in the model, ensuring its usability within the context of the model.

The function the algorithm seeks to find the least squares solution for is:

$$ln(m_{x,t}) = a_x + b_x k_t + \epsilon_{x,t} \tag{59}$$

$m_{x,t}$ is the central death rate at age x in year t. a_x is the general shape of mortality by age, and k_t is a time index of the general level of mortality across all age groups at time t. b_x describes the extent to which the mortality changes at a particular age when time elapses. $\epsilon_{x,t}$ is the error term that captures all age-dependent influences the model can not explain. In the model, $\epsilon_{x,t}$ has a normal distribution with mean zero and variance σ_ϵ^2 (The MathWorks, Inc., 2024).

I use European data published by the Actuarial Institute in 2022, also known as "het Actuarieel Genootschap" (AG) in Dutch (Actuarieel Genootschap, 2022). When fitting the data to equation (59) using the LC method, I use the whole data set published, with data starting in 1970 and ending in 2019. I sum up the data from both men and women as the OLG model does not distinguish between genders. Taking the data from Europe as a whole reduces the variance compared to taking data from the Netherlands. I calculate the mortality rate by dividing the number of deaths by the number of people exposed.

Following the Lee-Carter method, I fit equation (59) to this dataset. This results in the estimations of $\hat{b}_x$ and $\hat{a}_x$, which can be found in Appendix A.2, Figure 7. Furthermore, Figure 7 shows the seen value of k_t in the past. The exact method for finding the values of $\hat{b}_x$ and $\hat{a}_x$ is clearly outlined by The MathWorks, Inc. (2024).

Now I have valued $\hat{b}_x$ and $\hat{a}_x$, I only have to estimate the future values of k_t. k_t is a univariate time series, and because one can see a clear trend in the historical values of k_t, an autoregressive moving-average (ARMA) model is effective at forecasting k_t. However, the time series fitted to the ARMA model has to be stationary. In Figure 9 (Appendix A.2), one can see that k_t is not stationary as it has a decreasing trend. However, it can also be seen that the second-order difference is centred around zero, which suggests it to be stationary. The Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test confirms this.

The next step is to determine the orders of p and q, where p represents the number of time lags of the autoregressive part (AR), and q is the number of time lags of the moving-average part (MA). Figure 9 (Appendix A.2) displays the autocorrelation function (ACF) and the partial autocorrelation function (PACF), along with the corresponding "$\pm 1.96/\sqrt{T}$ 95 % confidence intervals" of the second order difference of k_t, where T is the number of periods (Kojvnikov, 2023a). For an ARMA(p,q) model, the ACF and PACF should show a direct or oscillating decay at lag q and p respectively (Kojvnikov, 2023a). As can be seen, neither the ACF nor the PACF decline gradually.

There are relatively few data points, implying a strong preference for a parsimonious model. Therefore, I do not consider models with lags greater than 2. As the choice for p and q is not straightforward from the ACF and PACF of the second order difference of k_t, I chose a model that minimizes the Bayesian Information Criterion (BIC), which prefers parsimonious models (Kojvnikov, 2023b). From Table 6, I can conclude that the BIC is the smallest for the ARMA model with $p = 1$ and $q = 1$ among the considered combinations of p and q.

Given the time series data $\Delta^2 k_t$ $t \in [0, 49]$ (data from 1970 to 2019), the ARMA model is given by:

$$\Delta^2 k_t = -0.0062 - 0.4182\Delta^2 k_{t-1} + \epsilon_t - 0.6763\epsilon_{t-1} \tag{60}$$

Plotting the original time series k_t with the fitted ARMA model confirms that the chosen model fits the original time series (Figure 10, Appendix A.2). I can now estimate future

Table 6: Information cirteria for different values of p and q, when fitting the ARIMA(p,q) to the second order difference of k_t

p	q	Variance	Residuals	Dim	BIC
1	1	1.703	1.735	3	184.154
1	2	1.787	1.817	3	186.571
1	[1,2]	1.703	1.735	4	188.066

values of $\hat{k}_t$. In combination with the time-independent values of $\hat{a}$ and $\hat{b}$, I can now forecast the mortality rates for all ages in the future. The survival rates are easily calculated as 1 minus the mortality probability at a particular age and year. In the OLG model, I split the population into three age groups. The probability of survival to age group 1 is one. I calculate the probability of survival from the age group 1 to the age group 2 as the product of the survival rates corresponding to the ages one up to 43. Likewise, I calculate the probability of survival from age group 2 to age group 3 as the product of the survival rates from 44 up to and including 67.

The projected survival rates, $\psi_{1,t}$ and $\psi_{2,t}$, can be seen in Figure 7 for $t = 0$ to $t = 3$, representing the years 2019 to 2079. Here, the survival rates at $t = 0$ are realised values. The 67 % upper bounds are used in Section 5.2 to show the sensitivity of the results to demographic changes.

Table 7: Forecasted survival rates age group s, including the 67% upper bound.

Age group	Forecast	$t = 0$	$t = 1$	$t = 2$	$t = 3$
$s = 1$	Forecast	0.980	0.985	0.989	0.992
	67% upper bound	0.980	0.982	0.981	0.979
$s = 2$	Forecast	0.894	0.897	0.894	0.886
	67% upper bound	0.894	0.924	0.952	0.973

Note. the value at $t = 0$ was the survival rate observed in 2019.

3.2.3 Total fertility rate projections

In the model, I use projections about the fertility rate made by Centraal Bureau van Statistiek (CBS). The total fertility rate (TFR) is calculated by dividing the number of children born of women at a certain age by the number of women that age. Hereafter, these age-dependent fertility numbers are summed up. From this calculation, one can already see that the timing of births strongly influences the total fertility rate. When the period for having children is delayed, the TFR is temporarily lower (CBS, 2020a).

In 2010, the total fertility rate observed was 1.80, after which it declined. This decline was particularly noticeable among couples in their twenties and young thirties, leading to a decrease in the number of babies born. However, the number of births among older women slightly increased. From this observation, CBS assumes that, although women started having children later in age, they will still have them later in life. In 2019, the observed fertility rate was 1.57. According to CBS's prognosis in 2019, the fertility rate will increase in the coming years to 1.7 in 2034, after which it stabilizes. In this prognosis, (future) net immigration is considered CBS (2020c).

As I do not distinguish between men and women in the OLG model, I divide the total fertility rate by two. Intuitively, this is explainable from equation (57). Moreover, in Section 5.2.4 I present how a low fertility rate affects the economy under both pension tax systems using the 67 % lower bound published by CBS (CBS, 2020c). All total fertility rates can be found in Table 8. [12]

Table 8: Projection total fertility rate

Scenario	$t = 0$	$t = 1$	$t = 2$
Baseline projection	0.785	0.85	0.85
67 % lower bound	0.785	0.765	0.73

3.2.4 Net migration

For determining the future *net* migration rates, I use, like for the fertility rate, the projection of the Centraal Bureau van Statistiek (CBS) published in 2019. CBS expects the yearly net migration to decrease in the upcoming years, from approximately 114,000 in 2019 to 39,000 in 2060. In the years 2019 to 2038, CBS expects a total net migration of 1,340,904. In the years 2039 up to and including 2058 this reduces to a number of 885,713. For the sake of simplicity, I assume all migrants are aged 20 when they enter the economy and have the same skill decomposition as natives. Moreover, I assume net migration to be stable after $t = 2$, so from 2059 onward.

As I normalise the adult population in the Netherlands in 2019 (13.486 million) to 1, I also divide the net migration with 13.486 million. This result in a baseline migration of $mr_1 = 0.10$, $mr_2 = 0.07$ and at last $mr_3 = 0.07$.

3.3 Preferences

Weight on leisure
γ: the weight on leisure is set such that agents under EET work on average one-third of their time. I use the calibration method of modularization and changing types using accepted packages available through *dynare*. This technique is fairly easy, and it is clearly explained by Wili Mutschler (2021). I create a new variable, l, calculated as:

$$l = \sum_{s=1,2} \sum_{j=1,2,3} \pi_j \frac{\Phi l^e_{s,j,t} + (1 - \Phi) l^s_{s,j,t}}{2} \tag{61}$$

Dynare finds the value of γ such that $l = \frac{1}{3}$ under EET in the steady state given the other calibration targets on productivity I introduce later in this Section. The calculations can

[12] By equation 57, the fertility rates at time $t = 3$ are not used to calculate the size of the age groups at time $t = 3$.

be found in Appendix A.4, Listing 18. This results in the following value of $\gamma \approx 0.329$. Note that the agent under EET can still decide on the distribution of the labour supply between the two working periods. As said in Section 3, I use the same value for γ in the model with TEE. The exact calculations can be found in Appendix A.4.

Consumption smoothing
σ: denotes the consumption smoothing coefficient of the agent. There is no consensus regarding an appropriate risk aversion coefficient σ value. σ ranges from 1.5 to 4 (Armstrong et al. 2015; Brissimis and Bechlioulis 2017). I set $\sigma = 1.5$ similar to Armstrong et al. (2015). In the sensitivity analysis (Section 6) the steady state results are shown for a 10% increase of σ.

Discount factor
β: denotes the discount factor, used in economic models to calculate the present value of future cash flow benefits. It represents the time value of money, reflecting the idea that agents typically value the same goods more now than in the future. Similarly to Armstrong et al. (2015), I set β equal to 0.99^{20}, similarly to Fehr et al. (2013), who set the time discount factor in a similar context equal to 0.985.

Productivity
ρ_j: the productivity rate of agent type j. As described in Section 3.1, there are three different agents with gross annual incomes of €24,552, €50,990 and at last €90,140.

The income of agent type j at working age is given by the product of the real wage rate, his productivity and his time spent working. I calibrate the productivity such that the average income during the working life of agent j equals the average gross income mentioned in Section 3.1. This ensures that the incomes are on the same scale as the tax brackets and the state pension. Moreover, then the income distribution in the model, although simplified, represents the real income distribution seen in the Netherlands in 2019. Similarly to determining γ, I determine the parameter values using modularization and changing types. ρ_j is set such that the average incomes of agents 1, 2 and 3 during working life in the steady state (representing 2019) are equal to 0.24552, 0.50990 and 0.90140 respectively. The average time spent working by agent j is calculated as:

$$- w_0 \times \rho_j \times \left(\Phi \frac{(l^e_{1,j,0} + l^e_{2,j,0})}{2} + (1 - \Phi) \frac{(l^s_{1,j,0} + l^s_{2,j,0})}{2} \right)$$

Please be aware that one euro symbolizes one hundred thousand euros in the model. The calculation can be found in Appendix A.4, Listing 18. This results in the following values of the productivities: $\rho_1 \approx 1.737$, $\rho_2 \approx 3.320$ and $\rho_3 \approx 6.333$.

3.4 Technology

Capital share of income
α: this parameter determines the capital share of income, which equals one minus the labour share of income. While the calculation of the labour share may seem straightforward, various challenges are associated with measuring the labour share of income (Guerriero, 2019). The conventional method calculates the labour share as the compensation of employees divided by the value added. Although this measure is widely used in

the literature, it underestimates the labour share as it disregards the contribution of the self-employed to the national income. In response, Guerriero (2019) proposes an adjustment based on the workforce composition. This distinction is especially important as in the OLG model, the economy's output includes the value of the self-employed.

$$\text{Labour share} = \frac{\text{Compensation of employees} \times \frac{\#\ \text{Total workforce-employers}}{\#\ \text{Number of employees}}}{\text{Value added}} \tag{62}$$

$$= \frac{388,403,000 \times \frac{9117-336,25}{7761,25}}{810,247,000 - 89,832,000 - 133,376,000} \approx 0.75$$

All numbers are in thousands. The value added is calculated as the GDP minus indirect taxes minus consumption fixed capital. Data on the compensation of employees and value added is taken from the National account of the Netherlands (2019c). Moreover, I use data from CBS regarding workforce composition (CBS, 2019a).[13] The resulting labor share is in line with the calculations of Guerriero (2019) with Dutch data from 1970 to 2015. The resulting capital share of income, α, is equal to $1 - 0.75 = 0.25$.

Total Factor Productivity
A: the technology level, also known as Total Factor Productivity. I set this parameter equal to 1 in accordance with Our World in Data (2019), a trusted data source in research and media, who measured the Total Factor Productivity of the Netherlands in 2019.

Physical depreciation of capital
δ: the depreciation rate of capital. I set the depreciation rate of capital similarly to Gomme and Lkhagvasuren (2013), who consider the substantially higher depreciation of equipment & software compared to that of market structures (housing). The final yearly depreciation rate of 7.18% is a weighted average of these two components, where the weights are given by the relative sizes of the two capital stocks (Gomme and Lkhagvasuren, 2013). Taking into account that a period in the model is equal to 20 years, this results in a depreciation rate given by $\delta = 1 - (1 - 0.0718)^{20} = 0.7747$.

3.5 Pension contribution, state pension and franchise

Pension contribution rate
ϕ: employees save a percentage, ϕ, of their pensionable base through mandatory workplace arrangements. I assume the percentage is equal for all ages, as envisaged in the new pension system. In the new pension system, this so-called flat contribution may be up to 30% of pensionable base (AON, 2021). In the model, I set the pension contribution rate to 20 %, resulting in a net replacement rate under the EET system in the steady state closely resembling to the realised net replacement rate seen the in the Netherlands in 2019 (85,8% and 80,0% respectively).

State pension (AOW)
$\tilde{b}$: the state pension. I assume all agents are entitled to the full state pension upon

[13] As I don't include work loss or disability in the model, the total workforce in this calculation is equal to the *working* labour force. The number of employers is equal to the self-employed with staff.

retirement. In 2019, the Dutch state pension (AOW) for singles was set at €15,157 per year (Belastingdienst, 2024). For married retirees (whose partner is also retired), the yearly AOW income was €10,339. According to CBS, throughout 2018, on average, 60.7% of the AOW recipients were married. The corresponding database is not updated for 2019. I assume that the percentage of married retired relative to the total retired population remains constant over time. Then I calculate $\tilde{b}_t$ as the average yearly state pension retired received in 2019. I thus establish:

$$\tilde{b}_t = (1 - 0.607)\frac{15,157}{100,000} + 0.607\frac{10,339}{100,000} \approx 0.12223 \tag{63}$$

Where again one euro represents 100,000 euros in the model.

Franchise
f: employees contribute to a pension scheme based on their earnings above the franchise level. As described in Section 2.1, the franchise is created as the savings in the second pillar are additional to the state pension. The franchise differs for married and unmarried employees and is calculated by multiplying the state pension with a factor of 100/75.[14] Therefore, I calculate the franchise as a factor 100/75 multiplied by the calculated state pension in the model: $f = \frac{100}{75}0.12223 \approx 0.16310$ (Belastingdienst, 2024).

3.6 Overview calibrated parameters

Except for the demographic projections, I calibrate all parameters to align with the baseline year 2019 in the Netherlands. Table 17 in Appendix A.1 summarizes all calibrated parameters and the respective calibration targets.

[14]The factor 100/75 is strictly seen only for so-called "middel-loon regelingen", the most common pension benefit arrangement in the Netherlands. Despite the fact that I have arranged the pension arrangement differently in the model (DC instead of DB), I use this franchise.

4 Calibration accuracy

In this Section, I briefly compare my model's steady-state results calibrated to the baseline year, 2019, to realised values in 2019. This comparison is possible as my model is designed to correspond to the EET tax system currently implemented in the Netherlands.

The results are steady-state results given the parameters calibrated to the baseline year, 2019.

Table 9: Steady-state results representing 2019 relative to realised values

Description	Model results	Realised 2019	Source
Real interest rate	4.14 %	0.36 %	DNB (2023a) & The World Bank (2023)
Ratio saving output	10.6%	14.76 %	CBS (2019c)
Ratio tax expenditure output	39.77 %	38.90 %	CBS (2019c)
Net replacement ratio pension	85.8 %	80.00 %	OECD (2019)
% self-employed with pension savings	15.6 %	10.3 %	Biesenbeek et al. (2020)
Output economy (billion €)	680,904	810,247	CBS (2019c)

The model shows a higher annual real interest rate (4.14%) than the real interest rate seen in reality in 2019 (0.36%).[15] However, this does not necessarily mean the model is not suited to answer my research question. According to Gomme and Lkhagvasuren (2013), the majority of macroeconomic literature is calibrated to a real return of 4%, justified by the idea that it represents the rough average of stock market returns of 7% and the return on risk-free bonds of 0.8%. Furthermore, the interest and inflation rates have varied a lot over time (De Nederlandsche Bank 2023a; The World Bank 2023). Therefore, it is not straightforward to compare the model's real interest rate to realised values. Disregarding whether the interest rate is too high, I can compare the impact of the different tax systems on the interest rate.

The ratio of savings to output (10.6%) is lower than the ratio seen in reality (14.76%). An explanation could be that agents only save for retirement in the model. In reality, however, agents save for various things, like unexpected expenditures or a bequest motive. Nevertheless, the net replacement ratio is slightly high compared to the value seen in the Netherlands (85.8% and 80.0% respectively).[16] Likely, the model's high real return on pension savings creates the difference. In the past 19 years, the actual real return of pension funds in the Netherlands was, on average, 2.73 % (Better Finance, 2022). I consciously calculate the net replacement rate instead of the gross replacement rate as I later compare the net replacement rates of the pension systems with the EET and TEE tax systems. Comparing the gross replacement rates would be unfair, given that taxes are collected at different stages in life.

[15] Looking at how equation (50) is constructed, I calculate the real interest rate seen in the Netherlands as the average interest rate Dutch banks charged non-financial corporations for loans lower than 0.25 million in 2019 ($2,96\%$), minus the annual inflation rate (2.6%).

[16] I calculate the net replacement ratio for agents by dividing the average disposable income during working life by the disposable income during pension. This net replacement ratio is a weighted average of all agents.

Moreover, the ratio of taxes paid to output is nearly the same as seen in reality. In the OLG model, firms make no profits and are not taxed. However, all earnings of the firms are spent on labour and capital, which are taxed. Likely, this indirectly offsets the simplification of not having corporate taxes.

Under EET, the steady-state results of the OLG model show that approximately 15.6 % of the self-employed saves in the third pillar, compared to 10.3% determined by DNB in 2020 (Biesenbeek et al., 2020). [17] Likely, the difference can be attributed to the simplification in the model, where self-employed are only distinguished by three levels of salaries. However, the difference is small. Therefore, we can conclude that the model gives a good representation of the behaviour of self-employed with respect to pension savings.

The model's economic output at $t = 0$ is lower than the Gross Domestic Product (GDP) observed in the Netherlands in 2019, potentially influenced by the absence of extremely wealthy individuals in the model and not accounting for export income. Nevertheless, the GDP is not unrealistic, as in 2016, the GDP was measured at 708,337 million euros. Hence, the model remains a reliable indicator of how the economy and the financial well-being of the typical work population respond to modifications in the tax system.

To conclude, although the model is simplified and I make numerous assumptions, the decisions made by the agents in the model and the tax paid align with reality. With this observation, the model is suitable to adjust slightly when changing the tax system to compare the effect of different tax schemes on the economy.

[17] $\frac{(pop_{0,1}\pi_3)(1-\Phi)}{(pop_{0,1}+pop_{0,2})(1-\Phi)} \approx 0.156\%$. Moreover, 10.3% is the percentage of self-employed who save in the third pillar but wo do not save in the second pillar.

5 Simulation results of the OLG model under the EET and TEE tax systems

In this Section, I show the simulation results of the OLG model. All results are local maxima, confirmed by the first-order derivatives of the Euler-equations.

Recall my research question: *What are the consequences of the Exempt-Exempt-Taxes (EET) and the Taxed-Exempt-Exempt (TEE) pension tax systems on the economy and welfare across generations and income groups amidst demographic changes in the context of the Netherlands?*

I divide the question into two sub-questions.

First: *What are the consequences of the Exempt-Exempt-Taxes (EET) and the Taxed-Exempt-Exempt (TEE) pension tax systems on the economy and welfare across generations and income groups in the context of the Netherlands?*

To answer this question, I show and analyse in Section 5.1 the results of the overlapping generations model with both the EET and the TEE systems, the last with different levels of subsidy τ_b. These are so-called steady-state results given the parameters calibrated in Section 3 to the baseline year, 2019 ($t = 0$). I refer to Section 2.7 for a detailed description of the steady state.

The Section is divided into three parts where I first examine in Section 5.1.1 the intergenerational effects of EET and TEE. Here I discuss how the different tax system affect the decisions made at different stages of life. I continue in Section 5.1.2 with analysing the implications on the distributional level. Lastly, I show the macroeconomic implications of the different tax systems. Building on the agent-level findings, I elaborate on the underlying causes of the seen macroeconomic differences.

Secondly: *To what extent are both tax systems robust in accommodating demographic change?*

In Section 5.2, I research how demographic changes affect the economy under EET and TEE, the latter with a subsidy level of 20 %.

In the OLG model, I normalise the adult population (13.456 million) to 1. Moreover, the (average) salaries, franchise and state pension are scaled in the model by a factor of 100,000. All results from the OLG model, except the prices and the well-being measurements, are multiplied by a factor of 1,345,600 million. Then. the results are in terms of 1,000 million.

5.1 Steady state simulation results: economic consequences of the EET and TEE tax systems

5.1.1 Intergenerational effects

In this Section, I present the results per age group to investigate the intergenerational consequences, separately for employees and self-employed. Table 10 displays the results of employees per age group, while Table 11 presents the results for self-employed agents.

The numbers differ substantially in magnitude as there are approximately four times more employees than self-employed.

Intergenerational effects of employees

Table 10: Aggregate results employees under EET and TEE per age group at time $t = 0$. All results are in terms of 1,000 million euros.

Employees

Variable	s	EET	TEE			
			$\tau_b = 0\%$	$\tau_b = 10\%$	$\tau_b = 20\%$	$\tau_b = 30\%$
Effective labour	1	482.4	603.6	594.8	587.5	581.4
	2	468.3	440.0	445.7	450.3	454.3
Savings	1	35.7	20.6	18.5	16.8	15.4
	2	28.9	12.9	12.1	11.4	10.8
Consumption	1	96.1	109.7	110.4	111.1	111.7
	2	96.4	85.1	87.3	89.3	91.1
	3	99.4	109.6	107.4	105.4	103.6
Income tax revenue	1	59.5	88.9	87.9	87.2	86.8
	2	59.8	60.7	62.1	63.4	64.5
	3	41.0	3.3	3.3	3.3	3.3

From Table 10, one can observe that, under EET, the amount of effective labour of the young adult employees is higher than of the middle-aged adults. However, the difference is small and can be mainly attributed to the larger young-aged group relative to the middle-aged group at $t = 0$. Examining the model with a TEE tax system reveals that under TEE, employees wish to front-load their effective labour. Agents recognise that a given amount of savings made at a younger working age contributes more to future pension income than identical savings made during an older working age, especially with the high interest rate seen under TEE. The young adult group's extra earnings are spent on savings in the second pillar and consumption. In absolute terms, the savings are still smaller under TEE than EET because of the elimination of delayed taxes. Nevertheless, these 'extra' savings of the young employees, combined with the higher interest rate under TEE, result in higher pension benefits. Contrarily, the decrease in labour supply of the middle-aged group decreases the corresponding consumption.

The tax revenue received from younger employees increases substantially under TEE, resulting from the increase of effective labour during the first stage of working life and the fact that savings are no longer tax-exempt. The tax revenue received from middle-aged employees increases slightly, indicating that the decrease in effective labour of middle-aged employees mostly offsets the extra tax revenue from abolishing tax-exempted savings. As expected, the tax revenue from retirees under TEE is substantially lower than under EET, as under TEE, only the state pension is taxed at retirement.

For higher subsidy levels under TEE, effective labour supply is shifted from the first to the second stage of working life. Moreover, the employees' pension savings decrease at higher subsidy levels. Note that the savings exclude the subsidies. As can be seen in equation (36), the government contributes with the subsidy to the mandatory pension savings of employees. Although the savings in the third pillar are also subsidised, it is not optimal for the employees to save more than mandatory. Despite the decrease in effective labour of the young employees for higher subsidy levels, their consumption increases. This shows that the subsidy increases disposable income after tax payment and mandatory pension savings, even when labour decreases. The consumption of middle-aged employees increases, resulting from the enlarged disposable income due to the subsidy and the extra time spent working. The subsidy does not increase consumption at retirement. Table 5.1.2 shows that the subsidy is no incentive for the employees to save more than mandatory due to the already high replacement rate. Therefore, the percentage of the pension savings with respect to the pensionable base remains constant over different subsidy levels. However, because employees save less in the first stage of life, in absolute terms, due to the decrease in time spent working (the savings which accrue the most returns), the consumption at retirement decreases.

Intergenerational effects of self-employed

Table 11 shows that self-employed wish to front-load their labour under EET, but particularly under TEE. However, the results are slightly misleading because only the highest-earning self-employed agent wishes to front-load their effective labour under both tax systems. As will be shown in Table 12, only the highest-earning self-employed agents save for a pension. The highest-earning self-employed recognise, like the employees, that savings earlier in life contribute more to their final pension than savings later in working life due to the high interest rate under TEE. As a result, the highest-earning self-employed increase their time spent working early in life substantially. As shown in Table 11, the total consumption of the retired self-employed is substantially higher than that of the young and middle-aged self-employed under TEE. Again, this is misleading as both the young and middle-aged agents do not save and, therefore, do not accrue pension in addition to the state pension. The high consumption at retirement, as seen in Table 11, is only due to the high consumption of the highest-earning self-employed at retirement. In Section 5.1.2 I elaborate on the decisions made by the basic and middle-earning self-employed. Moreover, the aggregate results per age group for the highest-earning self-employed are shown in Table 19 (Appendix A.2).

Table 11: Aggregate results self-employed under EET and TEE per age group at time $t = 0$. All results are in terms of 1,000 million euros.

Self-employed						
Variable	s	**EET**	**TEE**			
			$\tau_b = 0\%$	$\tau_b = 10\%$	$\tau_b = 20\%$	$\tau_b = 30\%$
Effective labour	1	113.7	139.5	139.5	139.5	139.5
	2	100.1	99.1	99.1	99.2	99.3
Savings	1	7.6	11.6	11.7	11.7	11.7
	2	**0**	**0**	**0**	**0**	**0**
Consumption	1	23.2	19.1	19.2	19.3	19.5
	2	23.8	21.8	22.0	22.1	22.2
	3	16.3	40.3	42.7	44.9	46.9
Income tax revenue	1	14.2	21.2	21.4	21.5	21.7
	2	15.1	13.3	13.4	13.6	13.7
	3	6.6	1.1	1.1	1.1	1.1

5.1.2 Distributional effects

Now that I have presented the results on the intergenerational level, I investigate the consequences of the different pension tax systems for all types of agents. Table 12 shows the average savings and time spent working during working life per type of agent. Moreover, the average consumption and income tax expenditure are shown over the agent's lifetime. Finally, Table 12 shows the net replacement rate and certainty equivalence. Recall that $j = 1$, $j = 2$ and $j = 3$ denote the basic, middle and highest-earning agent.

Under both EET and TEE, I can observe that the employed agent type j works, on average, more hours a day than the self-employed agent type j, regardless of the subsidy level under TEE. There is one exception: the highest-earning self-employed works more than the highest-earning employed under TEE. As I will discuss in Section 8, this tendency of the model under EET is not consistent with reality. As seen in Table 10, this increase is mainly due to the increased time spent working earlier in life, from the incentive to save extra, especially at younger ages under TEE.

The symbol * behind the savings in Table 12 denotes that the agent saves more than mandatory (which is zero for self-employees). Thus, under the EET system, there is an incentive to save more than the mandatory workplace pension for middle and highest-earning employees. This is beneficiary as these savings are deductible from the taxable income, and the tax rate at retirement is 19.45 % to earnings below €34,712 instead of 35.35 % at working age. Nevertheless, although not shown in this table, these extra savings are only made early in life, as the agents recognise that savings made earlier in life have a higher return. The budget constraint is likely too tight for the basic earning employee to save more than mandatory. Under TEE, no employed agents save more than the mandatory workplace pension, which is not surprising as the replacement ratios are already high with only the workplace pensions due to the high interest rate under TEE.

Under both EET and TEE, only the highest-earning self-employed save for a pension. The basic and middle-earning self-employed do not save under both EET and TEE. One could argue that under TEE, without subsidy, the incentive to save is missing as savings are not tax-exempted anymore like under EET. However, also under EET and TEE, with pension subsidy, which mitigates the gap between the tax rates at working life and retirement, the basic and middle-earning self-employees do not save. In the model of Armstrong et al. (2015), all agents are modelled as self-employed, with only the restriction that pension savings are non-accessible during working life. Here, all agents (modelled as self-employed) save for a pension under both tax systems. There are three explanations why, in contrast to the results of Armstrong et al. (2015), under my OLG model, the savings of the basic and middle-earning self-employees are zero under both tax systems.

In contrast to Armstrong et al. (2015), my model consists of employees and self-employed, of which the employees are the majority. As mentioned earlier, employees tend to work more than self-employees, especially under the TEE system. Likely, this is because employees have to work more to have a certain amount of consumption in addition to the mandatory pension savings. As the employees are the majority of the population, this suppresses the wage rate. Consequently, due to the lower wage rate, the self-employed choose fewer working hours at the cost of consumption and pension savings. The second explanation is the relatively high state pension seen in the Netherlands. In the model of Armstrong et al., the state pension was approximately 464 dollars a month (5,568 dollars a year). The relatively high state pension in the Netherlands of yearly €12,232 catches the loss of income during the last stage of life. Lastly, in the OLG model of Armstrong et al. (2015), agents do not include the probability of survival in their optimisation problem. Including this probability reduces the utility obtained through consumption at retirement.

If the highest-earning agents do not save for retirement, the drop in their consumption from working life to retirement (equal to the state pension) is substantial. Moreover, extra working time yields more earnings for the highest-earning self-employed than for the basic and middle-earning self-employed, incentivising them to work extra for savings (at a young age). As seen in Table 19 (Appendix A.2) higher subsidy levels increase the savings of the highest-earning self-employed minimally. Nevertheless, because the savings exclude the top-up subsidy, Table 12 shows that the net replacement rate increases substantially for higher subsidy levels. The increased consumption at retirement ultimately results in the higher consumption of the highest-earning self-employed on average over their lifetime.

Table 12: Results EET and TEE on distributional level at time $t = 0$. The time spent working and savings is calculated as the **average** time spent working during working life. Consumption and taxation are measured as the corresponding **average** throughout the three stages of life. The certainty level (ce) is calculated as $ce = U^{-1}(U(X))$, where X is the output of the simulation. $j = 1$, $j = 2$ and $j = 3$ correspond to the basic, higher and additional rate taxpayers. * donates there are savings made in the third pillar. Labour, savings, consumption and tax revenue are in terms of $\times$ 1,000 euros.

Variable		j	**EET**	**TEE**			
				$\tau_b = 0\%$	$\tau_b = 10\%$	$\tau_b = 20\%$	$\tau_b = 30\%$
Time spent working (%)	Employees		33.0	36.2	36.2	36.2	36.2
		2	36.0	38.0	37.9	37.8	37.7
		3	32.5	36.6	36.4	36.3	36.2
	Self-employed	1	29.2	28.8	28.8	28.9	28.9
		2	31.0	30.8	30.8	30.8	30.8
		3	32.1	39.2	39.2	39.2	39.2
Savings	Employees	1	1.8	1.4	1.3	1.2	1.2
		2	7.8*	4.3	3.9	3.6	3.4
		3	16.5*	7.7	7.1	6.5	6.0
	Self-employed	1	**0**	**0**	**0**	**0**	**0**
		2	**0**	**0**	**0**	**0**	**0**
		3	13.3*	20.4*	20.5*	20.6*	20.7*
Consumption	Employees	1	14.3	15.3	15.3	15.3	15.4
		2	26.7	27.2	27.3	27.3	27.3
		3	40.6	42.8	42.8	42.9	42.9
	Self-employed	1	12.2	11.5	11.5	11.6	11.6
		2	20.1	18.7	18.8	18.9	19.0
		3	40.6	68.1	71.4	74.5	77.3
Income tax	Employees	1	4.9	4.8	4.9	4.9	5.0
		2	12.4	11.2	11.2	11.2	11.3
		3	27.6	25.4	25.5	25.6	25.7
	Self-employed	1	4.1	3.6	3.6	3.6	3.7
		2	9.8	8.8	8.8	8.9	9.0
		3	27.6	28.1	28.3	28.5	28.7
Net replacement rate (%)	Employees	1	90.2	115.8	113.8	111.9	110.1
		2	92.9	112.6	110.1	107.8	105.6
		3	90.5	109.1	106.5	104.0	101.6
	Self-employees	1	67.8	73.7	73.3	72.9	72.5
		2	36.4	39.6	39.4	39.2	39.0
		3	90.3	291.1	309.1	325.5	340.6

Table 12 continued – Results EET and TEE on distributional level.

Variable		j	**EET**	**TEE**			
				$\tau_b = 0\%$	$\tau_b = 10\%$	$\tau_b = 20\%$	$\tau_b = 30\%$
Certainty equivalence	Employed	1	0.000 38	0.000 36	0.000 36	0.000 37	0.000 37
		2	0.000 65	0.000 61	0.000 62	0.000 63	0.000 64
		3	0.001 07	0.001 00	0.001 02	0.001 03	0.001 04
	Self-employed	1	0.000 37	0.000 35	0.000 35	0.000 35	0.000 35
		2	0.000 57	0.000 53	0.000 54	0.000 54	0.000 54
		3	0.001 08	0.000 88	0.000 89	0.000 90	0.000 92

Note. The savings exclude the top-up subsidy.

The average consumption is higher under TEE than under EET for all employees, created by the extra consumption early in life and at retirement, as shown in Section 5.1.1. The average consumption of the basic and middle-earning self-employed decreases due to the decrease in time spent working and, therefore, earnings. Contrarily, the consumption of the highest-earning self-employed agent increases substantially under TEE, mainly due to increased consumption during retirement.

One can observe that under TEE, all agents, except for the highest-earning self-employed, pay, on average, a lower amount of income tax throughout their lives than under EET. Under TEE, the pension contributions are taxed, whereas under EET, the pension benefits received at retirement, paid from the accumulated pension capital, are taxed. Consequently, a portion of the tax revenue in the EET system originates from the returns on pension capital, explaining the higher tax revenue observed under EET. This reasoning does not apply to the basic and middle-earning self-employed as they do not save for a pension. One can observe that the average income tax paid by these basic and middle-earning self-employed decreases from EET to TEE, explained by the lower earners resulting from the decrease in time spent working and the lower wage rate under TEE (seen in Table 13).

All in all, the certainty equivalent decreases for all employees when transitioning to TEE. Despite the reduction in average income taxes paid by, and the increase in average consumption of, the employees, the additional labour at a younger age creates substantial discomfort for the employees. The certainty equivalent of the self-employed decreases, irrespective of their income, with the highest-earning agent experiencing the most significant decline. Table 19 (Appendix A.2) shows that the extra earnings from the substantial increase in working time of the highest-earning self-employed early in life are primarily spent on pension savings. In the OLG model, agents prefer consumption and reject labour now rather than in the future. The substantial increase in consumption at retirement does not outweigh the discomfort of the decreased consumption and increased labour early in life, ultimately leading to a sharp decrease in the certainty equivalent. Moreover, the certainty equivalent of the basic and middle-earning self-employed is lower under TEE than under EET resulting from their lower consumption.

5.1.3 Macroeconomic effects

Table 13 displays the macroeconomic results, which are steady-state results given the parameters calibrated to the baseline year, 2019 ($t = 0$). The first column shows the parameter of interest, while the second column presents the results under the current EET tax system. The next four columns show the results under the TEE tax system with varying scenarios for the pension subsidy τ_b (to compensate for the disadvantage of higher taxes at working life than at retirement). Investment is the sum of savings and the subsidy, and therefore, under EET, the investment is equal to the savings. First, I compare the economic outcomes under the EET and the TEE tax system. Hereafter, I analyse how the introduction of a subsidy under TEE affects the economy under TEE.

Table 13: Macroeconomic results under EET and TEE at time $t = 0$. The latter with subsidy levels $\tau_b = 0$, $\tau_b = 0.10$, $\tau_b = 0.20$ and $\tau_b = 0.30$. All results, except the prices are in terms of 1,000 million euros.

Variable	**EET**	**TEE**			
		$\tau_b = 0\%$	$\tau_b = 10\%$	$\tau_b = 20\%$	$\tau_b = 30\%$
Macroeconomic aggregates					
Output	680.9	686.0	688.3	690.9	693.9
Capital	136.1	105.0	107.2	109.6	112.0
Effective labour	1 164.5	1 282.2	1 279.1	1 276.6	1 274.5
Consumption	355.2	385.6	389.0	392.1	395.0
Savings	72.2	45.1	42.3	39.9	37.9
Investment	72.2	45.1	46.5	47.9	49.3
Government	220.2	219.0	216.2	213.9	212.1
Prices					
Annual interest rate (%)	4.14	4.96	4.90	4.85	4.79
Wage rate	0.44	0.40	0.40	0.41	0.41
Taxation					
Taxes on Labour	196.2	188.6	189.3	190.2	191.1
Taxes on Consumption	74.6	81.0	81.7	82.3	83.0
Subsidy	0.0	0.0	4.2	8.0	11.4
Total taxes (net)	270.8	269.6	266.8	264.5	262.7

Note. Investment consists of savings plus the subsidy. The economy's output under all scenarios are checked to correspond to the Cobb-Douglas production function

The steady-state results show that the aggregate capital, the pension capital accrued from the investments and returns, is higher under EET than under TEE. Under EET, in contrast to under TEE, a proportion of the pension savings are delayed tax payments. These delayed savings are also available to be invested, and therefore, the aggregate investment are higer under EET. Moreover, the amount of effective labour (the productive labour supply) is higher under TEE than under EET.

Equation (51) shows that with a constant value of α and A, when K_t/L_t decreases, the interest rate increases. Under TEE, the amount of capital declines proportionally more than the effective labour increases relative to under EET. Therefore, the annual interest rate is higher under TEE than under EET. On the other hand, the wage rate is lower under TEE than EET resulting from the lower value of K_t/L_t under TEE (equation (52)). Intuitively, this is explainable: when the labour supply increases more than the resources/funds, the price of labour goes down. Despite the lower wage rate, moving to TEE results in an increase of time spent working by the employees, as shown in Section 5.1.2, ultimately increasing the effective labour.

The aggregate consumption is higher under TEE compared to EET. As shown in Table 10, young employees spent extra time working. Although savings are not deductible from taxable income under TEE, the extra time spent workig results in an increase of the consumption and saving for the young employees. The extra savings, combined with the high interest rate under TEE, result in higher consumption at retirement. Moreover, the consumption of the retired (highest-earning) self-employed increases substantially, as seen in Table 11.

Although the amount of effective labour is higher under TEE, the tax revenue from labour is greater under EET than under TEE. Under EET, the pension benefits at retirement, paid from the accumulated pension capital, are taxed. Therefore, a proportion of the tax revenue in the EET system is paid from accumulated return on capital, explaining the higher tax revenue under EET. Under TEE, the pension contributions are taxed, and the corresponding revenue is immediately spent on government expenses. However, whereas under EET the income tax revenue is higher, the tax revenue from consumption is higher under TEE, ultimately resulting in a slightly higher net tax revenue under EET relative to TEE (without saving subsidy).

Ultimately, the output, given by the equation $Y = C + G + \delta * K$ in the steady state, is higher under the TEE, without subsidy, than under the EET tax system. I refer to 2.4 for elaboration on the output calculation. Recall that the government expenditure (G) is calculated as the net tax revenue (T) minus the expenses on state pension. The expenses on state pensions are equal under both tax systems. Therefore, the government expenditures, like the net tax revenue, are slightly higher under EET than under TEE, without subsidy. Despite the higher government expenditures and the higher depreciation of capital (δK) under EET than under TEE, the higher consumption under TEE ultimately results in a higher output of the economy under TEE relative to EET.

As shown in Table 13, higher levels of pension subsidy under TEE, where the government contributes to the agents' pension accrual as a percentage of the individual's savings, results in an increase in the economy's output. As can be seen, the aggregate amount of savings decreases for higher levels of subsidy. Although savings above the mandatory pension savings are subsidised, the subsidy is no incentive for the employees to save more than mandatory (Table 12). Moreover, higher subsidy levels are an incentive for the employees to work less as it increases their disposable income after the mandatory savings, and therefore, the agents can maintain their consumption level when lowering their labour supply. Combined with the fact that the subsidy can be used to fulfil the mandatory pension savings, the savings of the employees decrease. As seen in Table 12, the subsidy is also no incentive for the basic and middle-aged earning self-employed to save. In contrast to the results of the employees, as shown in section 5.1.2, higher levels of subsidy levels are no incentive for the highest-earning agents to decrease their savings (Table 19, Appendix A.2). His pension savings remain relatively constant, increasing his net replacement rate. All in all, the amount of investment, equal to the savings plus the subsidy, increases for higher levels of subsidy, but only minimally.

Despite the decline in time spent working for higher subsidy levels, higher subsidy levels increases the consumption of all agents throughout their lives, except for the highest-earning self-employed, remains relatively constant. The subsidy effectively increases disposable income after taxation and (mandatory) pension savings. Nevertheless, the aggregate consumption increases considerately for higher subsidy levels, primarily due to the substantial increase of consumption of the retired highest-earning self-employed for higher levels of subsidy. The net tax revenue decreases for higher subsidy levels, implying that the higher tax revenue of consumption and labour do not compensate for the subsidy costs. As a consequence, with constant state pension expenditures, the government expenditures, consisting of net tax revenue and state pension expenditures, decrease for higher levels of subsidy. Nevertheless, because of the higher aggregate consumption and investment for higher subsidy levels, the economy's output increases for higher subsidy levels. With a 30% subsidy level, the economy's output under TEE is 693.9 billion euros relative to the economy's output under EET of 680,9 billion euros.

5.2 Perfect-foresight simulation results: the economy under the EET and TEE tax systems in the face of demographic changes

In addition to the research of Armstrong et al. (2015), I compare the trajectories of economic indicators under EET and TEE, the latter with a subsidy level of 20%, over time in the face of demographic changes. As Table 13 shows, the economic output under TEE increases for higher subsidy levels. Although the economic output is higher for a subsidy level of 30%, I investigate the TEE model with a 20 % subsidy level as I believe this is a realistic subsidy level the government would consider. As noted by Knell (2011): "The demographic challenge itself has two dimensions - people get older and they have fewer children". Therefore, I will show the trajectories of economic indicators under the baseline projection of the population and under the scenarios with high survival rates and low fertility. Section 5.2.1 describes how the size of the age groups changes under the three scenarios. In Sections 5.2.2, 5.2.3, and 5.2.4, I present the *perfect-foresight* simulations given the three demographic projections. I refer to Section 2.7 for a detailed explanation on perfect-foresight simulations. The simulation results at time $t = 0$ are equal to the steady-state results shown in Section 5.1. I present the percentage change of the economic indicators at time $t = 1$, $t = 2$ and $t = 3$ with respect to time $t = 0$. Lastly, I investigate in Section 5.2.5 the intergenerational risk-sharing consequences under EET and TEE. As mentioned in Section 2.1, I abstract from the DB elements currently implemented in the second (and third) pillar. In the Discussion (Section 8), I elaborate how this influences the results on intergenerational risk-sharing.

5.2.1 Development age groups under the three demographic scenarios

Given the projected values of fertility, survival and migration calibrated in Section 3.2, I can, using equation (1), calculate the development of the size of the age groups over time under the three scenarios demographic projections. The three demographic scenarios are:

1. Baseline scenario: with projected survival rates resulting from the Lee-Carter model and the projected fertility rates as stated by CBS.
2. High survival scenario: with the projected fertility rates and the 67% upper bound of survival probabilities .
3. Lower fertility scenario: with projected survival rates and the 67% lower bound of fertility.

Table 14: Size age groups and dependency rate under the baseline, high survival, and low fertility scenarios over time. The age group size is in terms of 1,000 million.

Scenario	**Variable**	$t = 0$	$t = 1$	$t = 2$	$t = 3$
Baseline	$pop_{1,t}$	4.72	5.05	5.24	5.40
	$pop_{2,t}$	4.63	4.63	4.98	5.18
	$pop_{3,t}$	4.14	4.14	4.22	4.62
	Dependency rate	2.26	2.34	2.42	2.29
High survival	$pop_{1,t}$	4.72	5.05	5.24	5.40
	$pop_{2,t}$	4.63	4.63	4.99	5.20
	$pop_{3,t}$	4.14	4.14	4.28	4.75
	Dependency rate	2.26	2.34	2.39	2.23
Low fertility	$pop_{1,t}$	4.72	5.05	4.81	4.46
	$pop_{2,t}$	4.63	4.63	4.98	4.76
	$pop_{3,t}$	4.14	4.14	4.22	4.62
	Dependency rate	2.26	2.34	2.32	1.99

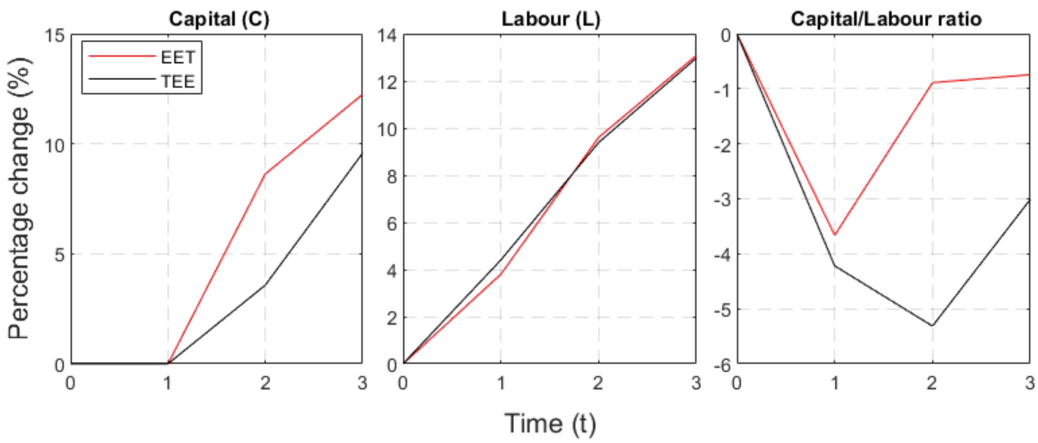
Note. The dependency rate is calculated as the size of the working age group divided by the retired. Moreover, $pop_{2,0} = pop_{2,1}$ as I calculate $pop_{2,0}$ as $pop_{1,0} * sr_{1,0}$, similar reasoning applies to $pop_{3,0} = pop_{3,1}$ (See Section 3.2.1). $pop_{1,1}$ is the same under all scenarios because fr_0 is the realised value seen in 2019 and equal in all scenarios. The survival rates at time $t = 0$ (observed in 2019) are equal under all scenarios.

As shown in Table 14, under under all scenarios, the dependency rate first increases after which it decreases. In the low fertility scenario, the dependency rate decreases to 1.99 at time $t = 3$ resulting mostly from the low fertility rate, in combination with the increasing survival rates over time, as seen in the baseline projections of survival.

5.2.2 Simulation results under the baseline demographic projection

Figure 3 displays the percentage changes in capital, effective labour and the capital/labour ratio (K/L) relative to time $t = 0$ under the baseline scenario with the projected survival and fertility rate. One can see that the amount of aggregate capital remains constant from the time $t = 0$ to $t = 1$, which can be explained by the measurement of the aggregate amount of capital K_{t+1}, at the beginning of time $t + 1$, and therefore determined at time t. Therefore, the amount of capital at $t = 1$ is exactly equal to the capital at time $t = 0$, stemming from the steady state results, under both EET and TEE. After $t = 1$, the aggregate amount of capital increases over time under both EET and TEE, but at a higher pace under EET. This steeper increase can be explained from both the higher incrase of investment under EET (Figure 4) and the capital returns on the delayed 'extra' investment resulting from the increasing working population. Moreover, Figure 3 shows that the amount of effective labour increases under both EET and TEE at a similar pace, resulting from the increased working population. Ultimately, because the increase in labour is steeper than the increase in capital from $t = 0$ to $t = 1$, the capital/labour ratio decreases under both pension tax systems from $t = 0$ to $t = 1$, with the highest decrease seen under TEE. From time $t = 1$ to $t = 2$, the ratio decreases further under TEE whereas it increases under EET. In the last time-lapse, from $t = 2$ to $t = 3$, the Capital/Labour ratio increases again, with the highest increase seen under TEE. Nevertheless, from $t = 0$ to $t = 3$, the Capital/Labour ratio returns almost to its initial value under EET whereas it decreases substantially under TEE. Figure 12 (Appendix A.2) shows the trajectories of the interest and the wage rate over time. The wage rate behaves opposite to the interest rate. Concluding from the Capital/Labour ratio, from $t = 0$ to $t = 3$, the interest rate increases most under TEE and subsequently, the wage rate decreases most under TEE.

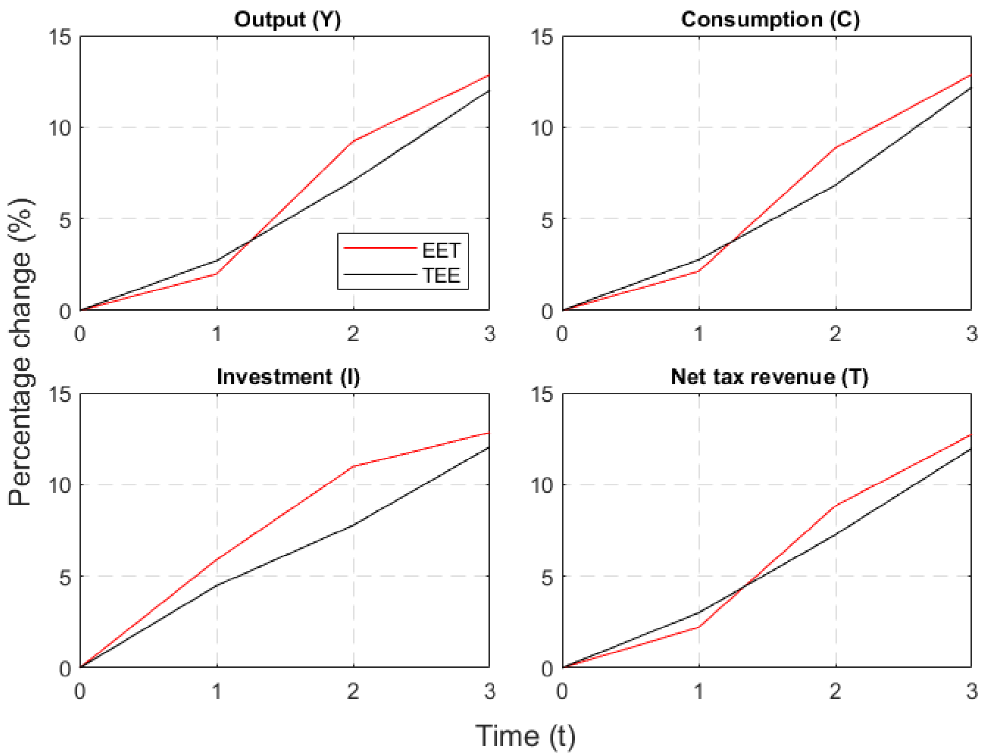
Figure 1: Labour and capital development under the baseline scenario with the projected fertility and survival rate.



Recall that the economy's output at time t (Y_t) is computed as the sum of aggregate consumption (C_t), government expenditures (G_t), and the *net* investment ($K_{t+1} - (1 - \delta)K_t$). The government expenditures is compromised of the net tax revenue (T_t) and state pension expenditures, for which the latter is equal under both tax systems. Moreover, the *net* investment is determined by the interest rate, survival rates and investment I_t.

Figure 4 shows that the investment increases at a higher pace under EET than under TEE. Note that savings in the third pillar can be more responsive to economic changes than the savings in the second pillar. As shown in Table 18 (Appendix A.2), the savings in the third pillar increase more over time under EET than under TEE under the baseline scenario. The aggregate consumption and net tax revenue increase over time, with the highest increase from $t = 0$ to $t = 3$ seen under EET. Overall, the economy's output increases under both the EET and TEE tax systems in all periods, with a slightly higher increase seen under EET from $t = 0$ to $t = 3$. However, the difference is small and neglectible over a period of 60 years. However, note that given the initial difference in output under EET and TEE, the output under TEE is higher at all times considered.

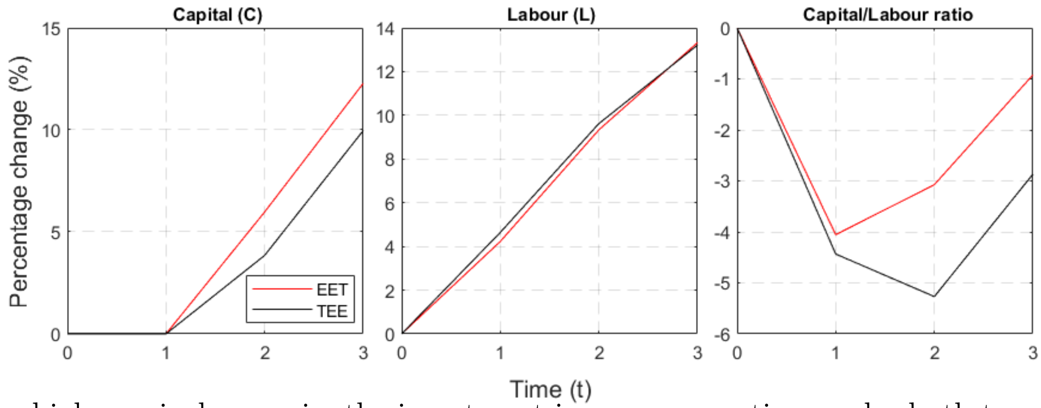
Figure 2: Economy under the baseline scenario with the projected fertility and survival rate.



5.2.3 Simulation results under the demographic projection with high survival

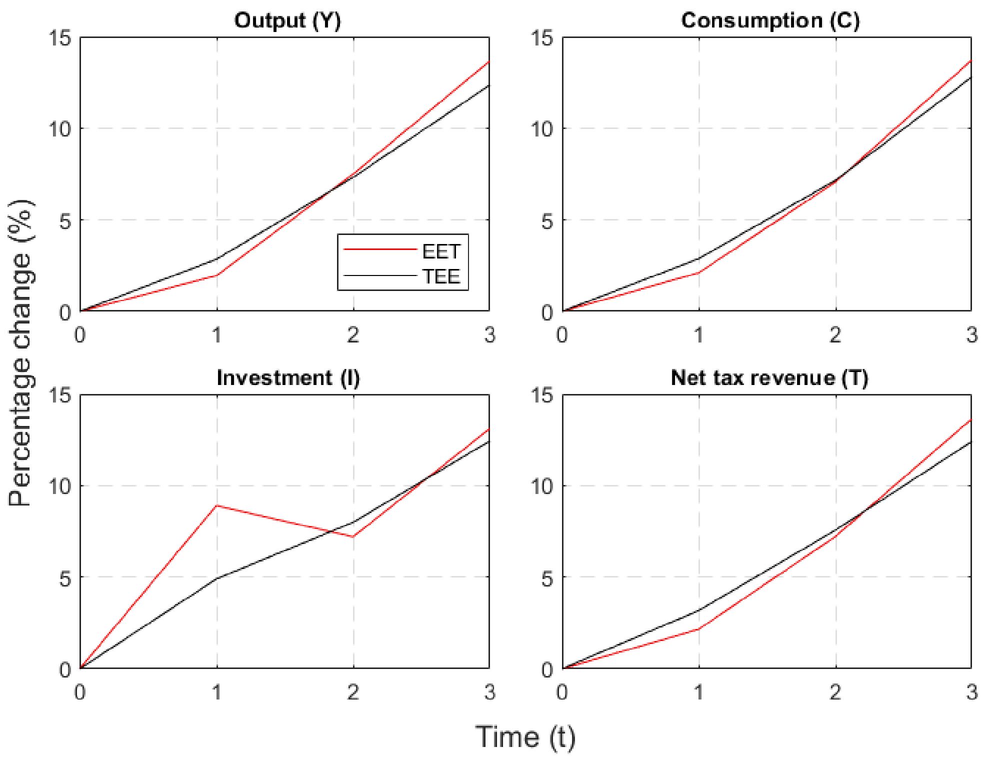
The trajectories of the economic indicators under the demographic scenario with the 67% upper bound of survival probability and the baseline projection of fertility are similar to those under the baseline scenario. Like under the baseline scenario, the aggregate capital increases more steeply under EET. Moreover, the higher probability of retirement induces the (employed) agents to work slightly more, increasing their savings and subsequently increasing the capital at a (slightly) faster pace than under the baseline scenario. Furthermore, resulting from the increased time spent working, the labour supply increases more steeply over time than under the baseline scenario. Ultimately, the Capital/Labour ratio decreases from $t = 0$ to $t = 3$ under both tax systems, with the highest decrease seen under TEE.

Figure 3: Labour and capital development under the scenario with the 67% upper bound of survival probability and the projected fertility rate.



In the high survival scenario, the investment increases over time under both tax systems, ultimately resulting in approximately the same increase of investment from $t = 0$ to $t = 3$ under EET and TEE. However, the investment under EET is more volatile due to greater changes in the savings in the third pillar under the high survival rate scenario (Table 18, Appendix A.2). Like under the baseline scenario, aggregate consumption and net tax revenue increase slightly more from $t = 0$ to $t = 3$ under EET than under TEE. Ultimately, the economic output increases slightly more under EET from $t = 0$ to $t = 3$ compared to under TEE. Nevertheless, the difference is small and can be neglected over a period of 60 years. However, because of the initial difference, the economy's output is still higher under TEE at all times considered.

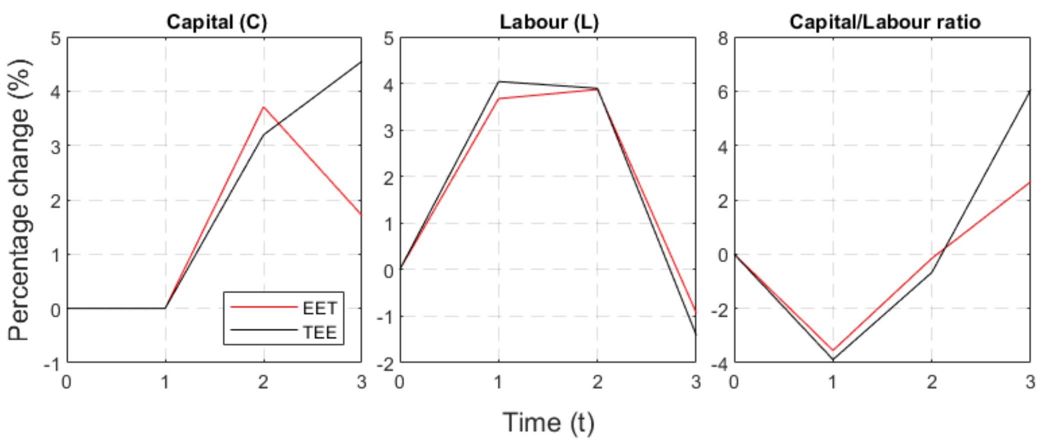
Figure 4: Economy under the scenario with the 67% upper bound of survival probability and the projected fertility rate.



5.2.4 Simulation results under the demographic projection with low fertility

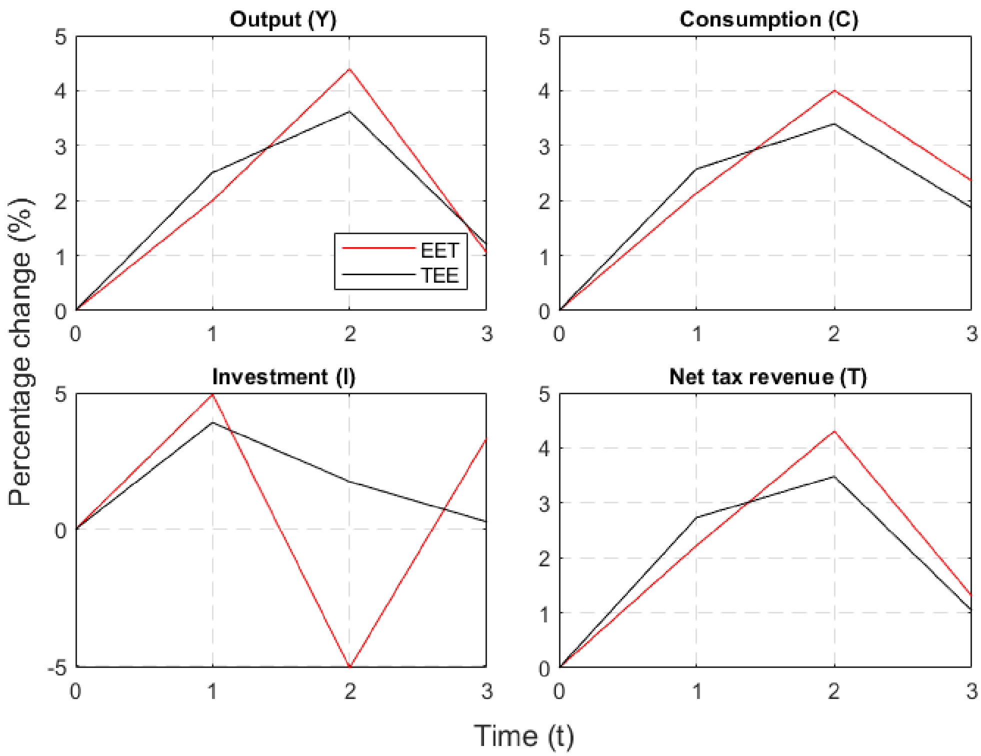
In this Section, I present the economy under EET and TEE in the face of the demographic scenario with the 67% lower bound of fertility. As shown in Table 14, the workforce first increases from $t = 0$ to $t = 1$, mainly due to the higher migration seen at $t = 1$. Then, from $t = 1$ to $t = 2$, the migration is partially offset by the declining fertility. From $t = 2$ to $t = 3$ the workforce declines due to the decreasing fertility rate. Figure 5 shows that, as a result, the labour supply increases from $t = 0$ to $t = 1$, remains relatively stable from $t = 1$ to $t = 2$, after which it declines steeply. Similarly to our observations in the baseline and high survival scenario, the amount of capital is equal at time $t = 0$ and $t = 1$. From the increase in the labour supply from $t = 0$ to $t = 1$ it is not surprising that the amount of capital (which is delayed by 1 period) increases from $t = 1$ to $t = 2$, as the savings in the second pillar increase with certainty if the labour supply (of employees) increases. From $t = 2$ to $t = 3$ the aggregate capital declines under EET, whereas it increases under TEE. The substantial decline in capital from $t = 2$ to $t = 3$ under EET results from the decline in investment from $t = 1$ $t = 2$ under EET as shown by Figure 6. Ultimately, the Capital/Labour rate first decreases from $t = 0$ to $t = 1$ after which it increases, with the highest increase seen under TEE. As a result, the interest rate decreases with respect to its initial value under both pension systems, with the highest decrease seen under TEE. (Figure 12, Appendix A.2).

Figure 5: Labour and capital development under the scenario with the 67% lower bound of fertility and the projected survival probability.



As shown in Figure 6, the percentage increase of investment at time $t = 3$ relative to its initial value is minimal under TEE. Under EET, the percentage increase of investment from $t = 0$ to $t = 3$ is higher, however, the investment is more volatile over time, as the savings in the third pillar are more responsive to demographic changes under EET than under TEE (Table 18, Appendix A.2). Moreover, net tax revenue and consumption increase under both tax systems from $t = 0$ to $t = 2$ after which it decreases from $t = 2$ to $t = 3$. Overall, net tax revenue and consumption increase from $t = 0$ to $t = 3$ more under EET than under TEE. Nevertheless, resulting from the substantial decrease in investment from $t = 1$ to $t = 2$ under EET, the percentage change in the economy's output over time relative to its initial value is approximately equal under the EET and TEE. However, given the initial higher value of the economy's output under TEE, the economy's output is still higher under TEE at all times considered.

Figure 6: Economy under the scenario with the 67% lower bound of fertility and the projected survival probability.



5.2.5 Intergenerational risk-sharing

As the second and third pillars are modelled as funded DC schemes, the intergenerational risk-sharing, often researched in the literature related to the impact of demographic changes on pension systems, is only affected through the first pillar. I investigate whether, as Don et al. (2013) suggest, the EET tax system provides the government better protection through the parallel movement of state pension expenditures and tax revenue. This protects the young workforce and future generations as, when the government can not pay the state pension anymore, the deficit will likely be covered by increasing taxation on the working generation or by increasing debt, which affect future generations.

Don et al. (2013) refer to scenarios where the government might be troubled to meet their state pension payments due to increased state pension benefit recipients and/or decreased income tax revenue, both resulting in a decreased dependency ratio. Answering whether one pension tax system can protect the government in these scenarios, one can look at the development of tax revenue under scenarios with high survival rates or low fertility, both resulting in a low dependency ratio. Table 15 presents the period-to-period percentage change for the two scenarios. As can be seen, the dependency ratio, the rate of workers to pensioners, decreases from $t = 2$ under the scenario with low fertility and from $t = 3$ in the scenario with high survival. Under both scenarios, the expenses of the state pension increase over time relative to time $t = 0$, with the highest increase seen in the scenario with high survival. Favorably, this increase in state pension expenses is partially met by increased tax revenue. Examining the tax revenue under both scenarios, I conclude, like Don et al. (2013) suggest, that under the scenario with high survival, the (net) tax revenue under EET indeed moves more closely with public pension expenses. However, the low fertility scenario shows mixed results. From $t = 1$ to $t = 2$, the tax revenue increases most under EET in correspondence with the increase of state pension expenditure. However, the tax revenue decreases from $t = 2$ to $t = 3$ more under EET than under TEE, likely resulting from the strong decrease of investment from $t = 1$ to $t = 2$ as seen in Figure 6, while the state pension expenditure increases. Therefore, I can not confirm the observation of Don et al. (2013) that EET protects the government better than TEE through the parallel movement of state pension expenditure and tax revenue in scenarios with a declining dependency rate.

Table 15: Period-to-period change under the EET and TEE system, the last with a 20% subsidy level. Under the 67% upper bound of survival probabilities, with a baseline fertility rate and for the 67% lower bound on fertility, with the baseline survival probability.

Scenario	**Variable**	$t = 0$	$t = 1$	$t = 2$	$t = 3$
High survival	Dependency ratio	0	3.45	2.18	−7.31
	State pension expenditure	0	0.00	3.25	10.07
	EET tax revenue	0	2.11	4.72	5.66
	TEE tax revenue	0	3.08	4.08	4.31
Low fertility	Dependency ratio	0	3.45	−0.88	−16.50
	State pension expenditure	0	0.00	1.95	8.78
	EET tax revenue	0	2.18	1.99	−2.97
	TEE tax revenue	0	2.66	0.72	−2.42

6 Sensitivity analysis with respect to the model assumptions

In addition to analysing the sensitivity of the results to demographic changes in Section 5.2, in this Section, I present a sensitivity analysis concerning some assumptions. The results are steady-state results and throughout the sensitivity analysis, all parameters are equal to the calibrated values except for the parameter in interest.

I show the results for a 10 % increase of σ. Note that the certainty equivalents for different values of σ are not comparable. Because α is generally set higher in related literature than the calibrated value, I show the results with a 10% increase of α (Armstrong et al. 2015; Fehr et al. 2013). Under EET, the weighted average spent working by all agents (employed and self-employed) was calibrated to be equal to 33.33 %. Under TEE, with a subsidy level of 20%, this increases to 36.84 %. It is questionable whether, in reality, individuals change the time they spend working in response to pension tax legislation. I show the results under TEE when I use the calibration method of modularisation and changing types to set the average time spent working by the agents equal to 33.33%. Lastly, I show the results under both tax systems when all agents accrue pension benefits through mandatory workplace pension schemes ($\Phi = 1$). Currently, only 10.3 % of the self-employed save for a pension (Biesenbeek et al., 2022). Suppose the pension market and the corresponding operators fail to address this pension coverage gap adequately. In that case, the government might obligate the self-employed to accrue for pension via arrangements in the second pillar.

Resulting from a 10% increase of α, the economy's output under EET decreases by approximately 8.6%, whereas the economy under TEE decreases by approximately 20.0%. Recall the Cobb-Douglas production function $Y = AK^{\alpha}L^{(1-\alpha)}$. α is the output *elasticity* of capital. If α is 25%, then if capital increases with 10%, then with equal labour, output increases with 2.5%. EET is more strongly 'capital based' compared to TEE: under EET the delayed taxation can be invested. If the economy is more capital based (higher value of α), then the decrease of the lower amount of capital is not counterbalanced by the higher effective labour under TEE relative to under EET. Therefore, it is unsurprising that a 10% increase of α results in a higher economic output under EET than under TEE.

For a 10 % increase of σ, the output under TEE remains higher than under EET, aligning the results in Section 5.1. The economy's output decreases for both tax systems under the higher value of σ, whereas the decrease is slightly steeper under EET. As shown in Figure 11 (Appendix A.2), the Cobb-Douglas utility received from the same amount of consumption and labour increases for a higher value of σ. However, in the optimisation problem, the utility received at middle age and retirement is discounted with the factors $\beta sr_{1,t}$ and $\beta^2 sr_{1,t} sr_{2,t}$ respectively, to account for the probability of survival and time preference. Therefore, the incentive to save for pension decreases and subsequently, investment decreases under both EET and TEE. However, the investment decreases more firmly under the EET tax system. This can be explained from several factors. First, the extra savings of the employed in the third pillar, only seen under EET, decreases. Secondly, saving under EET is more sensitive to saving behaviour due to the delayed taxes. Lastly, the subsidies received on pension savings from the government dampen the decrease in investment under TEE. The decrease in savings ultimately results in the observed decrease in consumption under both systems, as the return on capital used for additional consumption at retirement decreases. Overall, the steeper decrease in investment seen under EET results in a steeper decrease in the economic output under EET than under TEE.

As shown in Table 16, if the weighted average time spent working (l) is fixed for both EET and TEE to 33.33%, the economy's output is substantially higher under EET than under TEE.[18] Note that the results under EET do not change relative to the baseline steady-state results; here, the weighted labour is already fixed at 33.33%. Although the weighted time spent working is the same under both tax systems, the effective labour does not necessarily have to be. As we saw in Section 5.1.1, the labour supply under TEE is more front-loaded than under EET. The bigger size of the young adult age group with respect to middle-aged workers results in more effective labour under TEE than under EET. Nevertheless, fixing the time spent working under TEE substantially decreases the corresponding effective labour. Because of the elimination of delayed capital, the aggregate amount of capital is substantially lower under TEE than under EET. The output under TEE decreases substantially because when fixing the time spent working, the decrease in capital can not be compensated by an increase of effective labour (recall the Cobb-Douglas production function).

When all agents are obliged to save for pension ($\Phi = 1$), the economy's output is higher under the EET than under the TEE tax system. From Section 5, we know that self-employees save substantially less for retirement than employees, with one exception being the highest-earning self-employed under the TEE tax system (Table 12). When self-employed have to pay pension savings through workplace pension arrangements, they are modelled as employees, and therefore, intuitively, one would say that savings and, subsequently, investment increase under both tax systems. However, as seen in Table 16, the total investment only increases under EET. The disappearance of the considerable savings of the highest-earning self-employed, as seen under TEE, results in a drop in investment under TEE, higher than the increase in investment resulting from the increase in savings of the first-modelled-as basic and middle-earning self-employed. In contrast, the consumption under EET increases, resulting from the higher consumption at retirement. Consumption and investment under EET increase with respect to the baseline scenario, whereas under TEE, investment and consumption decrease with respect to the baseline scenario. Ultimately, this results in a higher economy's output under EET than under TEE.

To conclude, under a 10 per cent increase of σ, the economic output under TEE is higher than under EET, aligning the results in Section 5.1.3. However, in the following scenarios: 1) a 10 per cent increase of α, 2) fixing the labour under both EET and TEE to 33.33% and 3) obligating all agents to participate in mandatory workplace pension arrangements, the economy's output is higher under EET than under TEE. Moreover, under all investigated scenarios that the average certainty equivalent is consistently higher under EET than under TEE, aligning our results in Section 5.1.2.

[18] As seen in Section 3, $l = \sum_{s=1,2} \sum_{j=1,2,3} \pi_j \frac{\Phi l^e_{s,j,t} + (1-\Phi) l^s_{s,j,t}}{2}$.

Table 16: Steady state results for the model with the EET and TEE tax systems, each calculated using all calibrated parameters except for one variation of one parameter. I assume a subsidy level of 20% for the TEE tax system. All numbers except the *Well being* measurements and the *prices* are in terms of × 1000 million euros.

Variable	**Calibrated**		**110% α**[19]		**110% σ**		**Fixed labour ($l = \frac{1}{3}$)**		**Φ=1**	
	EET	TEE	EET	TEE	EET	TEE	EET	TEE[20]	EET	TEE
Macroeconomic aggregates										
Output	680.9	690.9	622.1	552.6	663.1	679.9	680.9	638.8	696.0	659.1
Capital	136.1	109.6	127.7	61.9	124.1	105.7	136.1	100.9	142.8	88.2
Effective labour	1 164.5	1 276.6	1 134.2	1 267.9	1 159.3	1 264.5	1 164.5	1 181.7	1 180.1	1 288.8
Consumption	355.2	392.1	333.5	350.4	351.4	388.3	355.2	368.1	361.7	386.5
Investment	72.2	47.9	64.5	23.3	65.5	45.9	72.2	43.5	80.0	38.7
Total taxes (net)	270.8	264.5	240.3	204.8	266.1	260.4	270.8	243.1	274.2	255.0
Prices										
Annual interest rate (%)	4.14	4.85	4.34	6.40	4.33	4.91	4.14	4.86	4.06	5.41
Wage rate	0.44	0.41	0.40	0.32	0.43	0.40	0.44	0.41	0.44	0.38
Well being										
Average net replacement (%)	85.8	113.9	93.2	141.9	88.5	114.9	85.8	114.2	89.0	127.4
Average certainty equivalent	0.00069	0.00066	0.00065	0.00058	0.00231	0.0022	0.00069	0.00040	0.00070	0.00066

Note. The certainty equivalents for different values of σ are not comparable.

7 Conclusion

This thesis studies the consequences of the Exempt-Exempt-Taxes (EET) and the Taxed-Exempt-Exempt (TEE) pension tax systems on the economy and welfare across generations and income groups amidst demographic changes in the context of the Netherlands.

I will first explain the macroeconomic results under EET and TEE without the introduction of a pension subsidy under TEE. Under TEE, analysing the steady-state results reveals that the economic output, computed in the steady state as $Y = C + G + \delta K$, is higher under TEE than under EET, amounting to 686,0 and 680,9 billion euros, respectively. Comparing the two tax systems, EET yields higher aggregate capital, compromised of pension savings and accumulated return, due to the delayed taxation. On the other hand, TEE shows increased effective labour. Despite the lower wage rate, TEE encourages employees, and the highest-earning agent to work more early in life, increasing both their consumption and savings in this period. They recognise that savings at a young age contribute more to their pension than similar savings later in working life, especially with the higher interest rate under TEE. As a result, consumption of the young and retired employees and highest-earning self-employed increase, the last resulting from the higher interest rate under TEE and the extra savings made at a younger age. All in all, the aggregate consumption under TEE is substantially higher than under EET. Therefore, also the tax revenue of consumption is higher under TEE than under EET. On the other hand, despite the higher effective labour under TEE, the income tax revenue is higher under EET. The deferred taxes under EET generate capital returns, increasing the income tax revenue. Ultimately, the government expenditures, compromising the tax revenue and state pension expenditures, are slightly higher under EET than under TEE. However, the increased consumption (C) under TEE offsets the higher amount of government expenditures (G) and depreciation of capital (δK) under EET, ultimately resulting in a higher economic output (Y) under TEE than under EET.

Under TEE, the individual's welfare level, as measured by the certainty equivalence, is lower than under EET, irrespective of the individual's characteristics. Drawbacks include the non-deductibility of tax savings during working life combined with the higher tax rate associated with the first tax bracket during working life than at retirement. Young adult employees and the highest-earning self-employed increase their working hours. However, the resulting burden is not counterbalanced by higher consumption earlier in life or higher pensions resulting from higher savings early in life and an elevated interest rate. Under both EET and TEE, the basic and middle-earning self-employed agents do not save for a pension, even under high pension subsidies under TEE. Several factors can explain this result. First, the majority of the population consists of employees who work more, especially under TEE, suppressing the wage rate. Secondly, the relatively high state pension in the Netherlands provides a safety net at retirement. Lastly, in the OLG model, agents incorporate survival probability, reducing the utility gained from retirement

[15]Under both the EET and TEE tax systems with 110% α, the values $l^e_{2,3,t}$, $l^s_{1,3,t}$ and $l^s_{2,3,t}$ are set such that the corresponding salary is equal to $\bar{y}_3$, because of the reasons explained in Section 2.7.

[20]Under the TEE tax system with fixed labour, the value $l^e_{2,3,t}$ is set such that the corresponding salary is equal to $\bar{y}_3$, because of the reasons explained in Section 2.7.

savings. With a decline in consumption during working life due to a lower wage rate and less time spent working, moving to TEE results in lower economic well-being of the basic and middle-earning self-employed, as measured by the certainty equivalent.

The economy's output under TEE rises when the government subsidises pension savings as a percentage of the individual's savings. This study shows that as the subsidy level increases, the economy's output under TEE also increases (investigated to a subsidy level of 30%). Employees can use the subsidy to meet their obligations in the second pillar. Therefore, the subsidy effectively boosts disposable income after mandatory pension payments, increasing consumption. However, the subsidy is no incentive for employees to save more than obligated, nor does it incentivise the basic and middle-earning self-employed to save for a pension. Nevertheless, the investment increases for higher subsidy levels, as the highest-earning self-employed do not decrease their savings, eventually increasing the investment, compromising the savings and the subsidy. Although net tax revenue decreases for higher subsidy levels, resulting from the higher subsidy costs, the higher levels of consumption and investment (/capital) result in an increase in overall economic output (639.98 billion euros with a 30% subsidy level). Moreover, higher subsidy levels increase the average economic well-being, as measured by the weighted average certainty equivalent. Nevertheless, the economic well-being remains higher under EET than under TEE with the highest investigated subsidy level of 30%.

Using three demographic scenarios, one with projected survival and fertility rates, one with high survival rates, and one with a low fertility rate, this thesis examines how the economy under both tax systems responds to demographic changes. The study shows that savings in the third pillar are more responsive to demographic changes under EET than under TEE. This adaptability of savings results in a slightly more stable interest and wage rate under EET compared to TEE across all demographic scenarios. In the scenarios with baseline survival and high survival rates, the population increases. As a result, consumption and net tax revenue increase under both tax systems, with a slightly higher increase seen under EET due to higher returns on savings/delayed taxes from the increased interest rate. Consequently, the economic output increases slightly more over time under EET than under TEE. However, over a period of 60 years, the difference is neglectable. Under the scenario with low fertility, the workforce first increases after which it declines steeply. As a result labour first increases after which it decreases. Moreover, capital, which is delayed by one period, only decreases the last period under EET. All in all, the decline in labour supply results in an increase in the wage rate and a decrease in the interest rate under both tax systems. The population decreasing after time $t = 2$ reduces consumption and net tax revenue under both tax systems, with the highest decrease seen under TEE. Nevertheless, resulting from the steep decline in investment at $t = 2$ under EET, the percentage change in the economy's output from $t = 0$ to $t = 3$ is similar under both tax systems in the scenario of low fertility. Overall, the economic output under EET and TEE react similarly to demographic changes with nuances on the impact of demographic changes on the components of the economy's output. However, TEE's initially higher economic output compared to EET leads to consistently higher output under TEE across all demographic scenarios for the next 60 years.

As the second and third pillars are modelled as funded DC schemes, intergenerational risk-sharing is primarily affected through the first pillar. When the government can not pay the state pension due to a declining dependency rate resulting from low fertility and/or ageing, the deficit will likely be covered by increasing taxation or debt, which affects the young workforce and future generations. Don et al. (2013) suggest that EET provides better protection for the government and future generations as tax revenue moves better along with the state pension expenditures. The study shows mixed results under the scenarios with high survival and low fertility, both with a declining dependency rate over time. Under the scenario with high survival probability, the tax revenue moves better along with the state pension expenditure under EET than under TEE. However, this parallel movement of state pension expenditure and net tax revenue is not seen under the scenario with low fertility.

Under all scenarios investigated in the sensitivity analysis, the overall economic well-being among the population, calculated as the average certainty equivalent, remains higher under EET compared to TEE. However, the sensitivity analysis shows that the conclusion, that the economic output is higher under TEE, with a subsidy level of 20%, than under EET, is dependent on the underlying assumptions. The study shows that if the weight on capital (α) increases with 10%, or when the average time spent working among the agents under TEE is fixed to 33.33% of their total time, just like under EET in the steady state, then the economic output under EET is higher than under TEE. Moreover, this thesis shows that when requiring self-employed to save through workplace pension arrangements, the economic output is higher under the EET system than under the TEE system. The legislation leads to an increase in investment under the EET system. In contrast, the investment under TEE decreases, resulting from the disappearance of the highest-earning self-employed with high pension savings. The increase in investment under EET also results in higher income tax revenue resulting from higher returns on delayed taxes. Despite the persistently higher consumption under TEE, the economic output under TEE is ultimately lower than under EET when all self-employed have to participate in workplace pension arrangements.

8 Discussion

In this Section, I discuss the most important findings of this thesis and relate them to existing literature. Moreover, I adress the findings of the sensitivity analysis and discuss all policies investigated in this thesis. Lastly, I discuss the limitations in the model possibly affecting the results.

The OLG model I use adapts the OLG model used by Armstrong et al. (2015). As mentioned in Section 2.4, the goods market clearing function Armstrong et al. (2015) use to calculate the economy's output, is inaccurate. Therefore I alter the goods clearing function such that the economic output is equal to the Cobb-Douglas production function, a condition that is not met by Armstrong et al. (2015). Moreover I alter the Government constraint under EET, to align with the budget constraints of the agents in retirement under EET. The economic output shown by Armstrong et al. (2015) under the EET and TEE tax systems is slightly overestimated. Nevertheless, the adaptation of the output calculation and government constraint under EET do not change their conclusion. Armstrong et al. (2015) conclude that the economy's output is lower under TEE than under EET, whereas my results suggest the opposite. The difference can be attributed to the introduction of employees in my model participating in mandatory workplace pension schemes. Under the model of Armstrong et al. (2015), the saving of working-aged agents drops strongly under TEE compared to EET, resulting from the decreased after-tax income under TEE, the drop of delayed taxation and the increased interest rate under TEE. Under my OLG model, the employees have no choice but to save also under TEE, at least what is mandatory. Moreover, the higher interest rate induces young employees and the highest-earning self-employed to work more, increasing their consumption early and later in life, the latter from the extra savings and higher interest rate. The total investment is still be higher under EET due to abolishing the delayed taxation. However, the increase in aggregate consumption under TEE eventually results in higher economic output under TEE than under EET.

Reflecting on the considerations of Don et al. (2013), one can make some conclusions or remarks. Don et al. (2013) state that without fiscal facilitation in the EET tax system, people would still build pensions through mandatory pension schemes. However, Don et al. (2013) recognise that fiscal facilitation is necessary in the third pillar, as otherwise, the barrier to save for retirement is too high. The steady-state results confirm that, unlike under EET, employees do not save in addition to workplace pension in the third pillar under TEE. However, consumption at retirement do not decrease, resulting from the higher interest rate and extra savings at a young age in the second pillar. Moreover, the highest-earning self-employed agent rather increases his savings in the third pillar under TEE compared to under EET likely resulting from the increased interest rate seen under TEE. Moreover, the basic and middle-earning self-employed save neither under EET and TEE. Overall, the results can not confirm, as Don et al. (2013) suggest, that the barrier to saving in the third pillar under TEE is too high. Secondly, Don et al. (2013) mention that high-income earners benefit slightly more from the tax facilitation under the EET tax system. This is true when looking at the benefits from a strict fiscal point of view. However, when looking at the well-being of the individuals, as measured by the certainty equivalence, the model shows that the economic well-being under TEE, relative to under

EET, decreases most for the highest-earning employed and self-employed. Lastly, the study can not confirm, as Don et al. (2013) suggest, that tax revenue moves better along with state pension expenditures.

Note that this study compares the economy under EET and TEE in steady state and in different scenarios of demographic development, it does not analyse a potential transition. A transition introduces complexities requiring further research. For example: how are the already-accumulated pension assets taxed? However, it does offer insights for policymakers in shaping a solid and efficient pension tax system. If the government aims to maximise the economic output, the steady state results suggest that TEE, with a subsidy level of 30%, yields the highest economic output. This is however at cost of a lower net tax revenue. Further research is advisable due to the mixed results in the sensitivity analysis. The higher economic output under TEE is primarily due to increased labour compensating for decreased capital assets under TEE. The sensitivity analysis suggests that with equal average working time under EET and TEE, the economic output is substantially higher under EET. Moreover, if the weight on capital (α) increases, the economy's output is higher under EET than under TEE. Further research on agents' reactions to tax policy changes and the relation between capital assets to GDP is necessary to understand the impact of the different tax systems on the economy. On the other hand, if the government aims to maximise the population's well-being, as measured by the average certainty equivalents of the population, the study suggests that under EET agents have the highest economic well-being across all income levels. The study shows that under all subsidy levels under TEE and all explored scenarios in the sensitivity analysis, the average economic well-being of the population is consistently higher under EET. This shows that the economic well-being of the population is not one-to-one related to economic output. It should be noted however, that although the certainty equivalent is a good indicator of the well-being of the population, it is neutral against changes in government expenses, which, in reality, can also affect agents' well-being.

The research emphasises the importance of policy measures, such as pension subsidies or mandatory pension savings for the self-employed. While a pension subsidy under TEE may lower net tax revenue, it effectively boosts consumption, ultimately increasing economic output. Moreover, legislating self-employed to save mandatory through workplace arrangements, yields a higher economic output under EET compared to TEE, with a 20% subsidy level, or TEE with also mandatory savings. As it also increases the average well-being of the population under EET, as measured by the certainty equivalence, mandating pension savings under the current EET tas system is an option worth further attention.

There are limitations in the model that could have affected the results. The OLG model is an example of a so-called closed economy. Therefore, the interest rate fluctuates under both tax systems due to the country's changing labour and capital formation to reach an equilibrium again. The Netherlands has in reality an open economy, and therefore it is questionable whether the interest rate, in reality, is different under EET and TEE. Therefore, the economic output under TEE tax system is likely overestimated, as the interest rate under TEE likely remains the same as under EET, thereby failing to mitigate the decline in investment. And if the interest and wage rates are responsive to changing consumption and labour patterns of agents, it is questionable whether these rates would change instantaneously in response to changes in consumption and labour. From a Keynesian view, the macroeconomy adjusts slowly after a change in fiscal policy because of sticky wages and prices: wages and prices that do not (instantly) respond to changes in demand (Whelan, 2016). Research incorporating an open economy and/or a sticky price setting where at least some firms are price setters can be useful to gain deeper understanding of the economic implications under both tax systems. Moreover, the model indicates that self-employed individuals generally work fewer hours than employees under the EET tax system. However, this observation contradicts evidence that, in particular, self-employees have longer working days (CBS, 2020b). This difference suggests a potential gap in capturing all relevant decision-making factors for both groups.

In the OLG model, I include redivision of capital of the agents among survival peers in the event of death. To ensure accuracy, I calculate the initial size of the age groups using the survival rates calculated in Section 3.2.2. I assume all agents that survived 60 years, to automatically turn 80 years, resulting in a relatively big size of the retired age group compared to reality (CBS, 2023). As a result, the aggregate capital might be overestimated as in reality there might be less accumulated capital and the effective labour might be underestimated as in reality there are in proportion less retired.

In this thesis I abstract from the DB elements currently implemented in the second (and third pillar). Therefore, the intergenerational risk-sharing effects of both tax systems in the currently implemented pension system might not be estimated correctly through the OLG model. The Netherlands has a multi-pillar system as described in Section 2.1. The publicly managed first pillar, financed from current tax revenue, follows the Pay-As-You-Go system and guarantees a flat state pension (PAYG DB) (Bonenkamp et al., 2017). The second pillar, consisting of occupational pensions, is funded and mostly based on Defined Benefit pension plans (funded DB). Moreover, most pension schemes in the second pillar employ the "doorsneesystematiek", where young workers pay more than what is actuarially fair for the pension benefits they built. Pension plans in the third pillar can employ the Defined Benefit and the Defined Contribution scheme. The funded DB scheme (with the "doorsneesystematiek") and the PAYG DB scheme are types of pension systems that share similarities: they guarantee benefits, and they both involve intergenerational risk-sharing (Bonenkamp et al., 2017). They are often cited as more vulnerable to unfavourable demographic trends, in contrast to PAYG DC and funded DC schemes (Bonenkamp et al., 2017). As I abstract from the DB elements seen under the currently implemented pension tax system, I could have overlooked on implications of the pension tax systems on the intergenerational risk-sharing. Nevertheless, the results represent the implications on intergenerational risk-sharing of the tax systems under the new pension system, where all pension plans in the second and third pillars are based on a funded DC system. [21]

[21] With some nuances, as in the new pension system, pension funds are obligated to maintain reserves for risk-sharing purposes.

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A Appendix

A.1 Calibrated parameters

Table 17: Parameter values

Description	Parameter	Value	Target
Tax system			
Consumption	τ_c	0.21	Ministerie van Financiën (2020)
Tax rate working agents j	(τ_1, τ_2, τ_3)	(0.3735, 0.3735, 0.4950)	""
Tax rate pensioners j	$(\tau_{ret1}, \tau_{ret2}, \tau_{ret3})$	(0.1945, 0.3735, 0.4950)	""
Excess tax rate working agents j	$(\tau_1^e, \tau_2^e, \tau_3^e)$	(0.3735, 0, 0.1215)	""
Excess tax rate pensioners j	$(\tau_{ret1}^e, \tau_{ret2}^e, \tau_{ret3}^e)$	(0.1945, 0.1790, 0.1215)	""
Income thresholds	$(\bar{y}_1, \bar{y}_2, \bar{y}_3)$	(0.0710,0.34712,0.68507)	Ministerie van Financiën (2020) and Financieel infonu (2021)
Demographics			
Share agents j	(π_1, π_2, π_3)	(0.3205, 0.3708, 0.3088)	Income distribution CBS (2019b)
Initial population, s $\in \{1, 2, 3\}$	$pop_{s,0}$	(0.3501, 0.3432, 0.3067)	In accordance to equation 1
Ratio employed	Φ	0.805	CBS (2019b)
Survival rate age group s	$\psi_{s,t}$	Table 7	Forecast Lee-Carter model
Fertility rate at $t \in \{1, 2, 3\}$	fr_t	(0.85, 0.85, 0.85)	Projection CBS (2020c)
Migration at $t \in \{1, 2, 3\}$	mr_t	(0.1, 0.07, 0.07)	Projection CBS (2020c)
Preferences			
Weight on leisure	γ	0.329	Average time working 1/3
Risk-aversion coefficient	σ	1.5	Armstrong et al. (2015)
Discount rate	β	0.82	Armstrong et al. (2015)
Productivity agents j	(ρ_1, ρ_2, ρ_3)	(1.737, 3.320, 6.333)	National account data (2019c) & average wages from CBS (2019b)
Technology			
Technology level	A	1	Our World in Data (2019)
Depreciation rate	δ	0.7747	Gomme and Lkhagvasuren (2013)
Capital share of income	α	0.25	National account data (2019c) and CBS (2019a)
Pension			
Pension contribution rate	ϕ	20%	Net replacement ratio under EET
State pension	$\tilde{b}$	0.122	Verzekeringsbank (2019)
Franchise	f	0.1631	State pension

A.2 Extra figures and tables

Figure 7: Mortality index over time and the estimated values of the age-specific values of $\hat{a}_x$ and $\hat{b}_x$

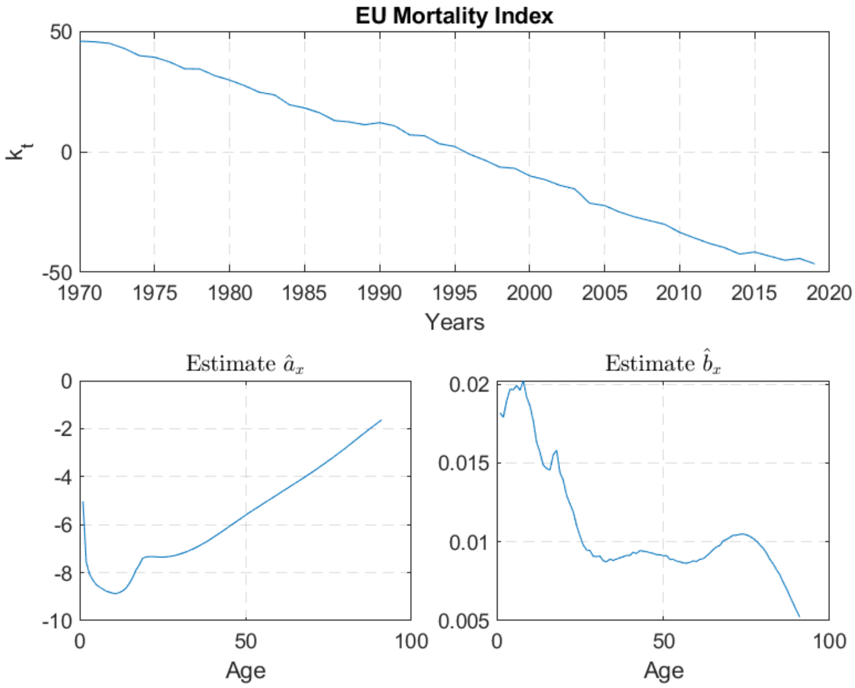

Figure 8: Mortality index k_t and the first and second order difference

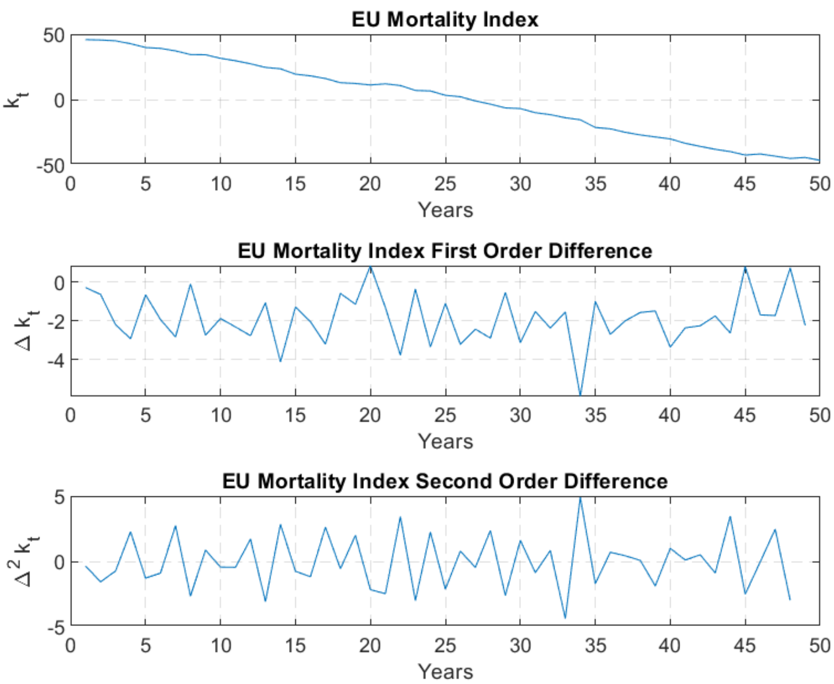

Figure 9: (Partial) Autocorrelation function second order difference EU mortality index (k_t)

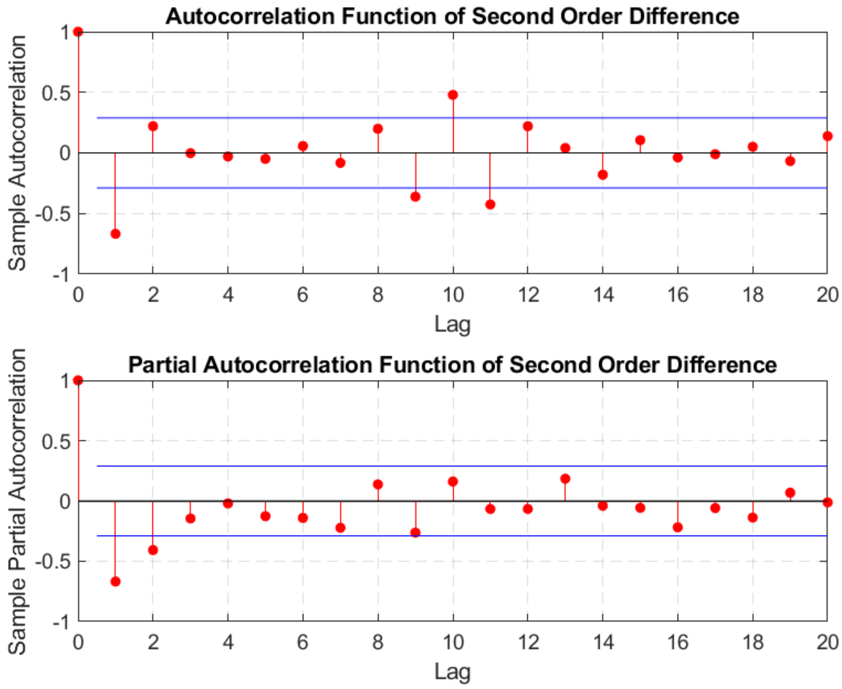

Figure 10: Time series k_t vs ARMA fit

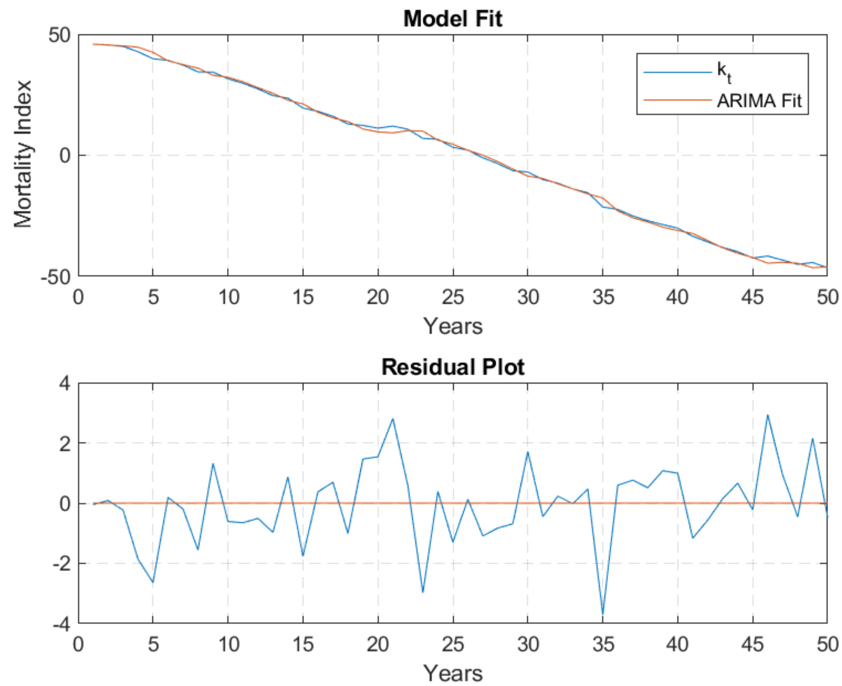

Figure 11: Cobb-Douglas utility with different values of σ

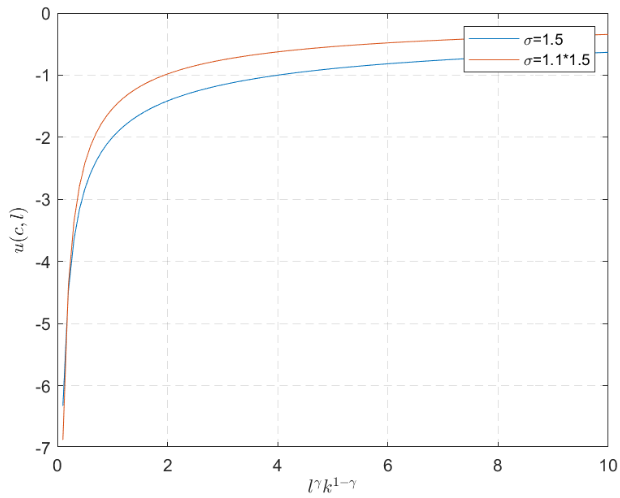

Table 19: Results high-earning self-employed under EET and TEE per age group at time $t = 0$. All results are in terms of 1,000 million euros.

Self-employed

Variable	**s**	**EET**	**TEE**			
			$\tau_b = 0\%$	$\tau_b = 10\%$	$\tau_b = 20\%$	$\tau_b = 30\%$
Effective labour	1	63.7	89.8	89.8	89.8	89.8
	2	51.0	50.4	50.4	50.5	50.5
Savings	1	7.6	11.6	11.7	11.7	11.7
	2	**0**	**0**	**0**	**0**	**0**
Consumption	1	10.4	7.4	7.5	7.5	7.5
	2	11.3	10.4	10.4	10.5	10.5
	3	11.1	35.2	37.5	39.7	41.8

Figure 12: The interest and wage rate under the three demographic scenarios

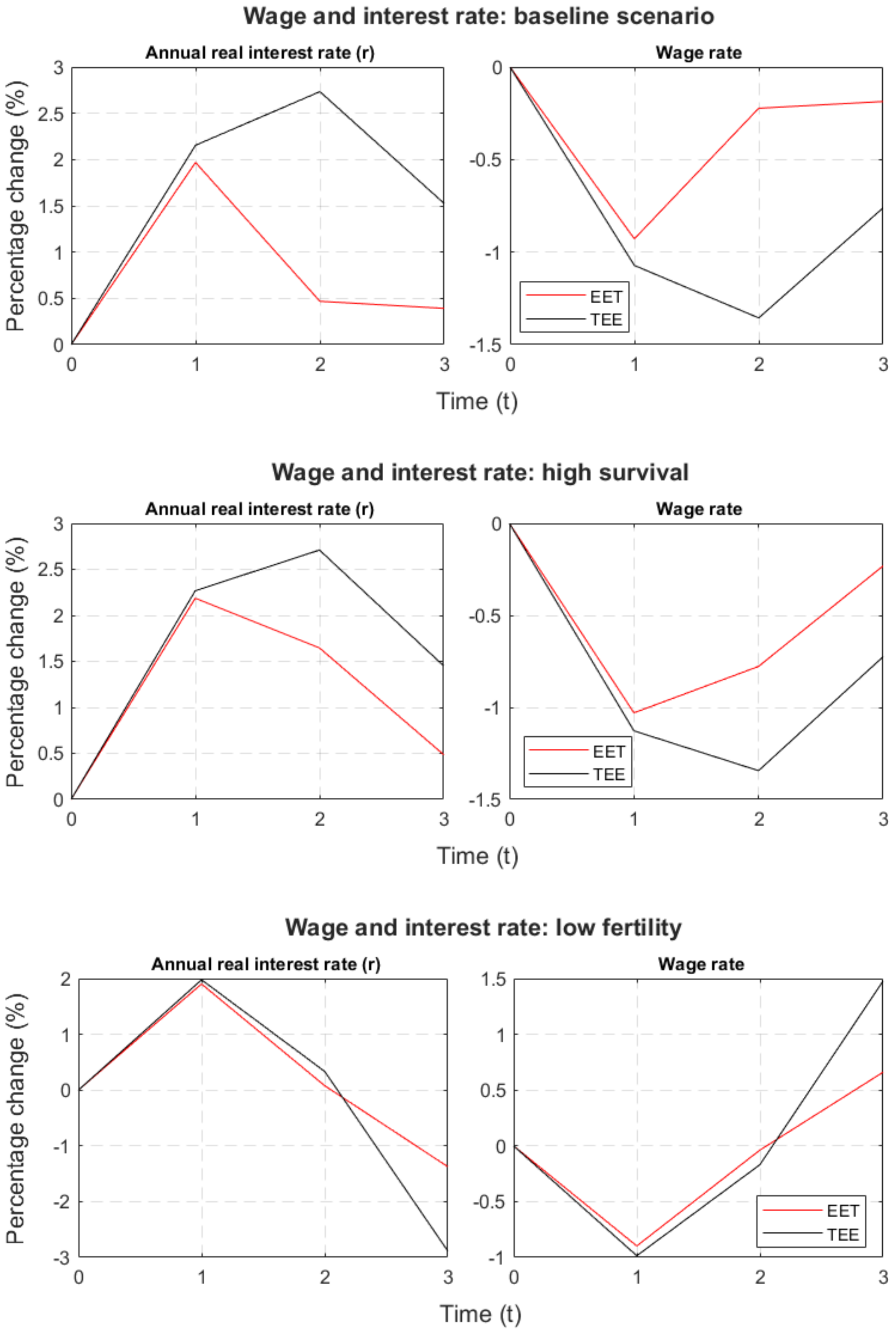

Table 18: Savings in the third pillar, by both employees and self-employed, under EET and TEE in the different demographic scenarios. The savings are in terms of 1,000 million euros.

Scenario	**Tax system**	$t = 0$	$t = 1$	$t = 2$	$t = 3$
Baseline	EET	13.39	16.17	15.77	15.16
	TEE	14.04	15.09	15.30	15.90
High survival	EET	13.39	18.03	13.84	15.25
	TEE	14.04	15.20	15.29	15.99
Low fertility	EET	13.39	15.58	7.70	15.60
	TEE	14.04	15.00	13.76	13.91

Note. The savings in the third pillar under TEE are only the results of savings from the highest-earning self-employed.

A.3 Proofs

Solving the individuals optimization problem, EET

Optimisation problem The agent maximizes their lifetime utility by adjusting the the capital he accumulates and the time he spents working. He maximizes over the capital accumulation instead of consumption or savings as consumption is directly related to savings, which subsequently determines capital accumulation. Under EET, the agent type j solves the following optimisation problem at time $t-1$. Under EET, agent type j's time t optimisation problem is:

$$\max_{\substack{k_{2,j,t+1},k_{3,j,t+2}\\ l_{1,j,t},l_{2,j,t+1}}} \psi_{0,t-1}\frac{(c_{1,j,t}^{\gamma}(1-l_{1,j,t})^{1-\gamma})^{1-\sigma}}{1-\sigma}+\beta\psi_{1,t}\frac{(c_{2,j,t+1}^{\gamma}(1-l_{2,j,t+1})^{1-\gamma})^{1-\sigma}}{1-\sigma} + \beta^2\psi_{1,t}\psi_{2,t+1}\frac{(c_{3,j,t+2}^{\gamma})^{1-\sigma}}{1-\sigma}$$

$$\begin{aligned}
\text{s.t.}\quad & (1+\tau_c)c_{1,j,t} = (1-\tau_{1,j,t})w_{j,t}l_{1,j,t} - (1-\tau_{1,j,t})\psi_{1,t}k_{2,j,t+1} + T_{1,j,t}\\
& (1+\tau_c)c_{2,j,t+1} = (1-\tau_{2,j,t+1})w_{j,t+1}l_{2,j,t+1} \quad .....\\
& \qquad - (1-\tau_{2,j,t+1})\left(\psi_{2,t+1}k_{3,j,t+2} - k_{2,j,t+1}(1+r_{t+1}-\delta)\right) + T_{2,j,t+1}\\
& (1+\tau_c)c_{3,j,t+2} = (1+r_{t+2}-\delta)k_{3,j,t+2}(1-\tau_{2,j,t+2}) + \tilde{b}(1-\tau_{3,j,t+2}) + T_{3,j,t+2}
\end{aligned}$$

For employees with **mandatory** pension savings, there are two additional constraints:

$$\begin{aligned}
& s_{1,j,t} = \psi_{1,t}k_{2,j,t+1} \geq \phi(w_{j,t}l_{1,j,t} - f)\\
& s_{2.j,t+1} = \psi_{2,t+1}k_{3,j,t+2} - k_{2,j,t+1}(1+r_{t+1}-\delta) \geq \phi(w_{j,t+1}l_{2,j,t+1} - f)
\end{aligned}$$

For self-employed with **voluntary** retirement savings, I have the following two additional constraints:

$$s_{1,j,t} = \psi_{1,t} k_{2,j,t+1} \geq 0$$
$$s_{2.j,t+1} = \psi_{2,t+1} k_{3,j,t+2} - k_{2,j,t+1}(1 + r_{t+1} - \delta) \geq 0$$

Employees The Lagrangian function for employees with **mandatory** savings of this optimisation problem is:

$$\begin{aligned} &L \\ &= \psi_{0,t-1} \frac{\left[\left(\frac{(1-\tau_{1,j,t})w_{j,t}l_{1,j,t} - (1-\tau_{1,j,t})\psi_{1,t}k_{2,j,t+1} + T_{1,j,t}}{(1+\tau_c)}\right)^{\gamma} (1 - l_{1,j,t})^{1-\gamma)}\right]^{1-\sigma}}{1-\sigma} \\ &+ \beta\psi_{1,t} \frac{\left[\left(\frac{(1-\tau_{2,j,t+1})w_{j,t+1}l_{2,j,t+1} - (1-\tau_{2,j,t+1})(\psi_{2,t+1}k_{3,j,t+2} - k_{2,j,t+1}(1+r_{t+1}-\delta)) + T_{2,j,t+1}}{(1+\tau_c)}\right)^{\gamma} (1 - l_{2,j,t+1})^{1-\gamma}\right]^{1-\sigma}}{1-\sigma} \\ &+ \beta^2\psi_{1,t}\psi_{2,t+1} \frac{\left[\frac{(1+r_{t+2}-\delta)k_{3,j,t+2}(1-\tau_{3,j,t+2}) + \tilde{b}(1-\tau_{3,j,t+2}) + T_{3,j,t+2}}{(1+\tau_c)}\right]^{\gamma(1-\sigma)}}{1-\sigma} \\ &- \lambda_{j,t}(\psi_{1,t}k_{2,j,t+1} - \phi(w_{j,t}l_{1,j,t} - f) - s_j^2) \\ &- \mu_{j,t+1}(\psi_{2,t+1}k_{3,j,t+2} - k_{2,j,t+1}(1 + r_{t+1} - \delta) - \phi(w_{j,t+1}l_{2,j,t+1} - f) - t_j^2) \end{aligned}$$

I assume $\psi_{0,t-1} = 1$.

Here the slack variables are s and t and the corresponding Lagrange multipliers are $\lambda_{t,j}$ and $\mu_{j,t+1}$. There are two options, either the slack variable is zero (which means the corresponding inequality constraint is active) or the Lagrange multiplier is zero (the constraint is inactive). I take deliberately s_j^2 and t_j^2 as to denote that they cannot be negative.

First order conditions with respect to labour supply for each j:

$$\begin{aligned} \frac{\partial L}{\partial l_{1,j,t}} &= \left[(c_{1,j,t})^{\gamma}(1 - l_{1,j,t})^{1-\gamma}\right]^{-\sigma} * \\ &\left[(1 - l_{1,j,t})^{1-\gamma}\gamma(c_{1,j,t})^{\gamma-1}\frac{(1 - \tau_{1,j,t})}{(1 + \tau_c)}w_{j,t} - (c_{1,j,t})^{\gamma}(1 - \gamma)(1 - l_{1,j,t})^{-\gamma}\right] + \lambda_{j,t}\phi w_{j,t} = 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial l_{2,j,t+1}} &= \beta\psi_{1,t}\left[(c_{2,j,t+1})^{\gamma}(1 - l_{2,j,t+1})^{1-\gamma}\right]^{-\sigma} \\ &\left[(1 - l_{2,j,t+1})^{1-\gamma}\gamma(c_{2,j,t+1})^{\gamma-1}\frac{(1 - \tau_{2,j,t+1})}{(1 + \tau_c)}w_{j,t+1} - (c_{2,j,t+1})^{\gamma}(1 - \gamma)(1 - l_{2,j,t+1})^{-\gamma}\right] + \mu_{j,t+1}\phi w_{j,t+1} = 0 \end{aligned}$$

First-order conditions with respect to capital stocks for each j become:

$$\begin{aligned} \frac{\partial L}{\partial k_{2,j,t+1}} &= -\frac{\psi_{1,t}(1 - \tau_{1,j,t})}{(1 + \tau_c)}(1 - l_{1,j,t})^{1-\gamma}\gamma(c_{1,j,t})^{\gamma-1}\left((c_{1,j,t})^{\gamma}(1 - l_{1,j,t})^{1-\gamma}\right)^{-\sigma} \\ &+\beta\psi_{1,t}(1-l_{2,j,t+1})^{1-\gamma}\gamma(c_{2,j,t+1})^{\gamma-1}((c_{2,j,t+1})^{\gamma}(1-l_{2,j,t+1})^{1-\gamma})^{-\sigma}\frac{(1 - \tau_{3,j,t+1})}{(1 + \tau_c)}(1+r_{t+1}-\delta) \\ &- \lambda_{j,t}\psi_{1,t} + \mu_{j,t+1}(1 + r_{t+1} - \delta) = 0 \quad (64) \end{aligned}$$

$$\frac{\partial L}{\partial k_{3,j,t+2}} = -\psi_{1,t}\beta\frac{\psi_{2,t+1}(1-\tau_{2,j,t+1})}{(1+\tau_c)}(1-l_{2,j,t+1})^{1-\gamma}\gamma(c_{2,j,t+1})^{\gamma-1}\left((c_{2,j,t+1})^{\gamma}(1-l_{2,j,t+1})^{1-\gamma}\right)^{-\sigma}$$
$$+\beta^2\psi_{1,t}\psi_{2,t+1}\gamma(c_{3,j,t+2})^{\gamma(1-\sigma)-1}(1+r_{t+2}-\delta)\frac{(1-\tau_{2,j,t+2})}{(1+\tau_c)}$$
$$-\mu_{j,t+1}\psi_{2,t+1} = 0 \quad (65)$$

Furthermore,

$$\frac{\partial L}{\partial \lambda_{t,j}} = \psi_{1,t}k_{2,j,t+1} - \phi(w_{j,t}l_{1,j,t} - f) - s_j^2 = 0$$
$$\frac{\partial L}{\partial \mu_{t,j}} = \psi_{2,t+1}k_{3,j,t+2} - k_{2,j,t+1}(1+r_{t+1}-\delta) - \phi(w_{j,t+1}l_{2,j,t+1} - f) - t_j^2 = 0$$
$$\frac{\partial L}{\partial s} = 2\lambda_{j,t}s = 0$$
$$\frac{\partial L}{\partial t} = 2\mu_{j,t+1}t = 0$$

Self-employed The Lagrangian function for **self-employed** with voluntary pension savings is:

$$L$$
$$= \psi_{0,t-1}\frac{\left[\left(\frac{(1-\tau_{1,j,t})w_{j,t}l_{1,j,t}-(1-\tau_{1,j,t})\psi_{1,t}k_{2,j,t+1}+T_{1,j,t}}{(1+\tau_c)}\right)^{\gamma}(1-l_{1,j,t})^{1-\gamma)}\right]^{1-\sigma}}{1-\sigma}$$
$$+\beta\psi_{1,t}\frac{\left[\left(\frac{(1-\tau_{2,j,t+1})w_{j,t+1}l_{2,j,t+1}-(1-\tau_{2,j,t+1})(\psi_{2,t+1}k_{3,j,t+2}-k_{2,j,t+1}(1+r_{t+1}-\delta))+T_{2,j,t+1}}{(1+\tau_c)}\right)^{\gamma}(1-l_{2,j,t+1})^{1-\gamma}\right]^{1-\sigma}}{1-\sigma}$$
$$+\beta^2\psi_{1,t}\psi_{2,t+1}\frac{\left[\frac{(1+r_{t+2}-\delta)k_{3,j,t+2}(1-\tau_{3,j,t+2})+\tilde{b}(1-\tau_{3,j,t+2})+T_{3,j,t+2}}{(1+\tau_c)}\right]^{\gamma(1-\sigma)}}{1-\sigma}$$
$$-\lambda_{t,j}(\psi_{1,t}k_{2,j,t+1} - s_j^2)$$
$$-\mu_{j,t+1}(\psi_{2,t+1}k_{3,j,t+2} - k_{2,j,t+1}(1+r_{t+1}-\delta) - t_j^2)$$

I assume $\psi_{0,t-1} = 1$.

Also here the slack variables are s and t and the corresponding Lagrange multipliers are $\lambda_{t,j}$ and $\mu_{j,t+1}$. There are two options, either the slack variable is zero (which means the corresponding inequality constraint is active) or the Lagrange multiplier is zero (the constraint is inactive).

First order conditions with respect to labour supply for each j:

$$\frac{\partial L}{\partial l_{1,j,t}} = \left[(c_{1,j,t})^{\gamma}(1-l_{1,j,t})^{1-\gamma}\right]^{-\sigma}$$
$$\left[(1-l_{1,j,t})^{1-\gamma}\gamma(c_{1,j,t})^{\gamma-1}\frac{(1-\tau_{1,j,t})}{(1+\tau_c)}w_{j,t} - (c_{1,j,t})^{\gamma}(1-\gamma)(1-l_{1,j,t})^{-\gamma}\right] = 0$$

$$\frac{\partial L}{\partial l_{2,j,t+1}} = \beta\psi_{1,t}\left[(c_{2,j,t+1})^{\gamma}(1-l_{2,j,t+1})^{1-\gamma}\right]^{-\sigma}$$
$$\left[(1-l_{2,j,t+1})^{1-\gamma}\gamma(c_{2,j,t+1})^{\gamma-1}\frac{(1-\tau_{2,j,t+1})}{(1+\tau_c)}w_{j,t+1} - (c_{2,j,t+1})^{\gamma}(1-\gamma)(1-l_{2,j,t+1})^{-\gamma}\right] = 0$$

These two first order equations yield two **labour-leisure Lagrange multiplier equations**, namely:

$$\frac{(1-\tau_{1,j,t})}{(1+\tau_c)}w_{j,t} = \frac{1-\gamma}{\gamma}\frac{c_{1,j,t}}{1-l_{1,j,t}} \tag{66}$$

$$\frac{(1-\tau_{2,j,t+1})}{(1+\tau_c)}w_{j,t+1} = \frac{1-\gamma}{\gamma}\frac{c_{2,j,t+1}}{1-l_{2,j,t+1}} \tag{67}$$

First order conditions with respect to capital stocks for each j become:

$$\begin{aligned}\frac{\partial L}{\partial k_{2,j,t+1}} &= -\frac{\psi_{1,t}(1-\tau_{1,j,t})}{(1+\tau_c)}(1-l_{1,j,t})^{1-\gamma}\gamma(c_{1,j,t})^{\gamma-1}\left((c_{1,j,t})^{\gamma}(1-l_{1,j,t})^{1-\gamma}\right)^{-\sigma}\\ &+\beta\psi_{1,t}(1-l_{2,j,t+1})^{1-\gamma}\gamma(c_{2,j,t+1})^{\gamma-1}((c_{2,j,t+1})^{\gamma}(1-l_{2,j,t+1})^{1-\gamma})^{-\sigma}\frac{(1-\tau_{2,j,t+1})}{(1+\tau_c)}(1+r_{t+1}-\delta)\\ &- \lambda_{j,t}\psi_{1,t} + \mu_{j,t+1}(1+r_{t+1}-\delta) = 0\end{aligned} \tag{68}$$

$$\begin{aligned}\frac{\partial L}{\partial k_{3,j,t+2}} &= -\psi_{1,t}\beta\frac{\psi_{2,t+1}(1-\tau_{2,j,t+1})}{(1+\tau_c)}(1-l_{2,j,t+1})^{1-\gamma}\gamma(c_{2,j,t+1})^{\gamma-1}\left((c_{2,j,t+1})^{\gamma}(1-l_{2,j,t+1})^{1-\gamma}\right)^{-\sigma}\\ &+ \beta^2\psi_{1,t}\psi_{2,t+1}\gamma(c_{3,j,t+2})^{\gamma(1-\sigma)-1}(1+r_{t+2}-\delta)\frac{(1-\tau_{3,j,t+2})}{(1+\tau_c)}\\ &- \mu_{j,t+1}\psi_{2,t+1} = 0\end{aligned} \tag{69}$$

Furthermore,

$$\frac{\partial L}{\partial \lambda_{t,j}} = \psi_{1,t}k_{2,j,t+1} - s_j^2 = 0$$
$$\frac{\partial L}{\partial \mu_{t,j}} = \psi_{2,t+1}k_{3,j,t+2} - k_{2,j,t+1}(1+r_{t+1}-\delta) - t_j^2 = 0$$
$$\frac{\partial L}{\partial s} = 2\lambda_{j,t}s = 0$$
$$\frac{\partial L}{\partial t} = 2\mu_{j,t+1}t = 0$$

Solving the individuals optimization problem, TEE

Optimisation problem Under TEE, agent type j's time t optimisation problem is:

$$\max_{\substack{k_{2,j,t+1},k_{3,j,t+2}\\ l_{1,j,t},l_{2,j,t+1}}} \frac{(c_{1,j,t}^{\gamma}(1-l_{1,j,t})^{1-\gamma})^{1-\sigma}}{1-\sigma} + \beta\psi_{1,t}\frac{(c_{2,j,t+1}^{\gamma}(1-l_{2,j,t+1})^{1-\gamma})^{1-\sigma}}{1-\sigma} + \beta^2\psi_{1,t}\psi_{2,t+1}\frac{(c_{3,j,t+2}^{\gamma})^{1-\sigma}}{1-\sigma}$$

$$\text{s.t.} \quad (1+\tau_c)c_{1,j,t} + \frac{\psi_{1,t}k_{2,j,t+1}}{1+\tau_b} = w_{j,t}l_{1,j,t}(1-\tau_{1,j,t}) + T_{1,j,t}$$
$$(1+\tau_c)c_{2,j,t+1} + \frac{\psi_{2,t+1}k_{3,j,t+2} - k_{2,j,t+1}(1+r_{t+1}-\delta)}{1+\tau_b} = w_{j,t+1}l_{2,j,t+1}(1-\tau_{2,j,t+1}) + T_{2,j,t+1}$$
$$(1+\tau_c)c_{3,j,t+2} = (1+r_{t+2}-\delta)k_{3,j,t+2} + \tilde{b}(1-\tau_{ret1}) + T_{3,j,t+2}$$

For an **employed** agent with mandatory pension savings, the two additional constraints are:

$$s_{1,j,t}(1+\tau_b) = \psi_{1,t}k_{2,j,t+1} \geq \phi(w_{j,t}l_{1,j,t} - f)(1-\tau_{retj})$$
$$s_{2.j,t+1}(1+\tau_b) = \psi_{2,t+1}k_{3,j,t+2} - k_{2,j,t+1}(1+r_{t+1}-\delta) \geq \phi(w_{j,t+1}l_{2,j,t+1} - f)(1-\tau_{retj})$$

For a **self-employed** agent with voluntary retirement savings, the two additional constraints are:

$$s_{1,j,t}(1+\tau_b) = \psi_{1,t}k_{2,j,t+1} \geq 0$$
$$s_{2.j,t+1}(1+\tau_b) = \psi_{2,t+1}k_{3,j,t+2} - k_{2,j,t+1}(1+r_{t+1}-\delta) \geq 0$$

Employees The Lagrangian function for employees with **mandatory** savings of this optimisation problem is:

$$\begin{aligned}
&L\\
&= \psi_{0,t-1}\frac{\left[\left(\frac{w_{j,t}l_{1,j,t}(1-\tau_{1,j,t})}{1+\tau_c} - \frac{\psi_{1,t}k_{2,j,t+1}}{(1+\tau_b)(1+\tau_c)} + \frac{T_{1,j,t}}{1+\tau_c}\right)^{\gamma}(1-l_{1,j,t})^{1-\gamma)}\right]^{1-\sigma}}{1-\sigma}\\
&+ \beta\psi_{1,t}\frac{\left[\left(\frac{w_{j,t+1}l_{2,j,t+1}(1-\tau_{2,j,t+1})}{1+\tau_c} - \frac{\psi_{2,t+1}k_{3,j,t+2}-k_{2,j,t+1}(1+r_{t+1}-\delta)}{(1+\tau_b)(1+\tau_c)} + \frac{T_{2,j,t+1}}{1+\tau_c}\right)^{\gamma}(1-l_{2,j,t+1})^{1-\gamma}\right]^{1-\sigma}}{1-\sigma}\\
&+ \beta^2\psi_{1,t}\psi_{2,t+1}\frac{\left[\frac{(1+r_{t+2}-\delta)k_{3,j,t+2}+\tilde{b}(1-\tau_{ret1})+T_{3,s,t+2}}{(1+\tau_c)}\right]^{\gamma(1-\sigma)}}{1-\sigma}\\
&- \lambda_{t,j}\left(\psi_{1,t}k_{2,j,t+1} - \phi(w_{j,t}l_{1,j,t} - f)(1-\tau_{retj}) - s_j^2\right)\\
&- \mu_{j,t+1}\left(\psi_{2,t+1}k_{3,j,t+2} - k_{2,j,t+1}(1+r_{t+1}-\delta) - \phi(w_{j,t+1}l_{2,j,t+1} - f)(1-\tau_{retj}) - t_j^2\right)
\end{aligned}$$

I assume $\psi_{0,t-1} = 1$.

Here the slack variables are s and t and the corresponding Lagrange multipliers are $\lambda_{t,j}$ and $\mu_{j,t+1}$. There are two options, either the slack variable is zero (which means the corresponding inequality constraint is active) or the Lagrange multiplier is zero (the constraint is inactive). I take deliberately s_j^2 and t_j^2 as to denote that they cannot be negative.

First order conditions with respect to labour supply for each j:

$$\frac{\partial L}{\partial l_{1,j,t}} = \left[(c_{1,j,t})^{\gamma}(1-l_{1,j,t})^{1-\gamma}\right]^{-\sigma}$$
$$\left[(1-l_{1,j,t})^{1-\gamma}\gamma(c_{1,j,t})^{\gamma-1}\frac{(1-\tau_{1,j,t})}{(1+\tau_c)}w_{j,t} - (c_{1,j,t})^{\gamma}(1-\gamma)(1-l_{1,j,t})^{-\gamma}\right] + \lambda_{j,t}\phi w_{j,t}(1-\tau_{retj}) = 0$$

$$\frac{\partial L}{\partial l_{2,j,t+1}} = \beta\psi_{1,t}\left[(c_{2,j,t+1})^{\gamma}(1-l_{2,j,t+1})^{1-\gamma}\right]^{-\sigma}$$
$$\left[(1-l_{2,j,t+1})^{1-\gamma}\gamma(c_{2,j,t+1})^{\gamma-1}\frac{(1-\tau_{2,j,t+1})}{(1+\tau_c)}w_{j,t+1} - (c_{2,j,t+1})^{\gamma}(1-\gamma)(1-l_{2,j,t+1})^{-\gamma}\right]$$
$$+ \mu_{j,t+1}\phi w_{j,t+1}(1-\tau_{retj}) = 0$$

First-order conditions with respect to capital stocks for each j become:

$$\frac{\partial L}{\partial k_{2,j,t+1}} = -\left((c_{1,j,t})^{\gamma}(1-l_{1,j,t})^{1-\gamma}\right)^{-\sigma}\gamma(c_{1,j,t})^{\gamma-1}(1-l_{1,j,t})^{1-\gamma}\frac{\psi_{1,t}}{(1+\tau_b)(1+\tau_c)}$$
$$+ \beta\psi_{1,t}\left((c_{2,j,t+1})^{\gamma}(1-l_{2,j,t+1})^{1-\gamma}\right)^{-\sigma}\gamma(c_{2,j,t+1})^{\gamma-1}(1-l_{2,j,t+1})^{1-\gamma}$$
$$\frac{1}{(1+\tau_b)(1+\tau_c)}(1+r_{t+1}-\delta) - \lambda_{j,t}\psi_{1,t} + \mu_{j,t+1}(1+r_{t+1}-\delta) = 0$$

$$\frac{\partial L}{\partial k_{3,j,t+2}} = = -\beta\psi_{1,t}\left((c_{2,j,t+1})^{\gamma}(1-l_{2,j,t+1})^{1-\gamma}\right)^{-\sigma}\gamma(c_{2,j,t+1})^{\gamma-1}(1-l_{2,j,t+1})^{1-\gamma}\frac{\psi_{2,t+1}}{(1+\tau_b)(1+\tau_c)}$$
$$+ \beta^2\psi_{1,t}\psi_{2,t+1}\gamma(c_{3,j,t+2})^{\gamma(1-\sigma)-1}\frac{1}{(1+\tau_c)}(1+r_{t+2}-\delta)$$
$$- \mu_{j,t+1}\psi_{2,t+1} = 0$$

Furthermore,

$$\frac{\partial L}{\partial \lambda_{t,j}} = \psi_{1,t}k_{2,j,t+1} - \phi(w_{j,t}l_{1,j,t} - f)(1-\tau_{retj}) - s_j^2 = 0$$
$$\frac{\partial L}{\partial \mu_{t,j}} = \psi_{2,t+1}k_{3,j,t+2} - k_{2,j,t+1}(1+r_{t+1}-\delta) - \phi(w_{j,t+1}l_{2,j,t+1} - f)(1-\tau_{retj}) - t_j^2 = 0$$
$$\frac{\partial L}{\partial s} = 2\lambda_{j,t}s = 0$$
$$\frac{\partial L}{\partial t} = 2\mu_{j,t+1}t = 0$$

Self-employed The Lagrangian function for **self-employed** with voluntary pension savings is:

$$
\begin{aligned}
&L \\
&= \psi_{0,t-1} \frac{\left[\left(\frac{w_{j,t} l_{1,j,t}(1-\tau_{1,j,t})}{1+\tau_c} - \frac{\psi_{1,t} k_{2,j,t+1}}{(1+\tau_b)(1+\tau_c)} + \frac{T_{1,j,t}}{1+\tau_c}\right)^{\gamma} (1-l_{1,j,t})^{1-\gamma)}\right]^{1-\sigma}}{1-\sigma} \\
&+ \beta\psi_{1,t} \frac{\left[\left(\frac{w_{j,t+1} l_{2,j,t+1}(1-\tau_{2,j,t+1})}{1+\tau_c} - \frac{\psi_{2,t+1} k_{3,j,t+2} - k_{2,j,t+1}(1+r_{t+1}-\delta)}{(1+\tau_b)(1+\tau_c)} + \frac{T_{2,j,t+1}}{1+\tau_c}\right)^{\gamma} (1-l_{2,j,t+1})^{1-\gamma}\right]^{1-\sigma}}{1-\sigma} \\
&+ \beta^2 \psi_{1,t}\psi_{2,t+1} \frac{\left[\frac{(1+r_{t+2}-\delta) k_{3,j,t+2} + \tilde{b}(1-\tau_{ret1}) + T_{r,j,t+2}}{(1+\tau_c)}\right]^{\gamma(1-\sigma)}}{1-\sigma} \\
&- \lambda_{t,j}\left(\psi_{1,t} k_{2,j,t+1} - s_j^2\right) \\
&- \mu_{j,t+1}\left(\psi_{2,t+1} k_{3,j,t+2} - k_{2,j,t+1}(1+r_{t+1}-\delta) - t_j^2\right)
\end{aligned}
$$

Also here the slack variables are s and t and the corresponding Lagrange multipliers are $\lambda_{t,j}$ and $\mu_{j,t+1}$. There are two options, either the slack variable is zero (which means the corresponding inequality constraint is active) or the Lagrange multiplier is zero (the constraint is inactive).

First order conditions with respect to labour supply for each j:

$$
\frac{\partial L}{\partial l_{1,j,t}} = \left[(c_{1,j,t})^{\gamma}(1-l_{1,j,t})^{1-\gamma}\right]^{-\sigma} \left[(1-l_{1,j,t})^{1-\gamma}\gamma(c_{1,j,t})^{\gamma-1}\frac{(1-\tau_{1,j,t})}{(1+\tau_c)} w_{j,t} - (c_{1,j,t})^{\gamma}(1-\gamma)(1-l_{1,j,t})^{-\gamma}\right] = 0
$$

$$
\frac{\partial L}{\partial l_{2,j,t+1}} = \beta\psi_{1,t}\left[(c_{2,j,t+1})^{\gamma}(1-l_{2,j,t+1})^{1-\gamma}\right]^{-\sigma} \left[(1-l_{2,j,t+1})^{1-\gamma}\gamma(c_{2,j,t+1})^{\gamma-1}\frac{(1-\tau_{2,j,t+1})}{(1+\tau_c)} w_{j,t+1} - (c_{2,j,t+1})^{\gamma}(1-\gamma)(1-l_{2,j,t+1})^{-\gamma}\right] = 0
$$

These two first-order equations yield two **labour-leisure Lagrange multiplier equations**, namely:

$$
\frac{(1-\tau_{1,j,t})}{(1+\tau_c)} w_{j,t} = \frac{1-\gamma}{\gamma}\frac{c_{1,j,t}}{1-l_{1,j,t}} \tag{70}
$$

$$
\frac{(1-\tau_{2,j,t+1})}{(1+\tau_c)} w_{j,t+1} = \frac{1-\gamma}{\gamma}\frac{c_{2,j,t+1}}{1-l_{2,j,t+1}} \tag{71}
$$

I assume $\psi_{0,t-1} = 1$.

First-order conditions with respect to capital stocks for each j become:

$$
\begin{aligned}
\frac{\partial L}{\partial k_{2,j,t+1}} = &- \left((c_{1,j,t})^{\gamma}(1-l_{1,j,t})^{1-\gamma}\right)^{-\sigma}\gamma(c_{1,j,t})^{\gamma-1}(1-l_{1,j,t})^{1-\gamma}\frac{\psi_{1,t}}{(1+\tau_b)(1+\tau_c)} \\
&+ \beta\psi_{1,t}\left((c_{2,j,t+1})^{\gamma}(1-l_{2,j,t+1})^{1-\gamma}\right)^{-\sigma}\gamma(c_{2,j,t+1})^{\gamma-1}(1-l_{2,j,t+1})^{1-\gamma} \\
&\frac{1}{(1+\tau_b)(1+\tau_c)}(1+r_{t+1}-\delta) - \lambda_{j,t}\psi_{1,t} + \mu_{j,t+1}(1+r_{t+1}-\delta) = 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial k_{3,j,t+2}} =& -\beta\psi_{1,t}\left((c_{2,j,t+1})^{\gamma}(1-l_{2,j,t+1})^{1-\gamma}\right)^{-\sigma}\gamma(c_{2,j,t+1})^{\gamma-1}(1-l_{2,j,t+1})^{1-\gamma}\frac{\psi_{2,t+1}}{(1+\tau_b)(1+\tau_c)} \\
&+\beta^2\psi_{1,t}\psi_{2,t+1}\gamma(c_{3,j,t+2})^{\gamma(1-\sigma)-1}\frac{1}{(1+\tau_c)}(1+r_{t+2}-\delta) \\
&-\mu_{j,t+1}\psi_{2,t+1}=0
\end{aligned}
$$

Furthermore,

$$
\begin{aligned}
\frac{\partial L}{\partial \lambda_{t,j}} &= \psi_{1,t}k_{2,j,t+1}-s_j^2=0 \\
\frac{\partial L}{\partial \mu_{t,j}} &= \psi_{2,t+1}k_{3,j,t+2}-k_{2,j,t+1}(1+r_{t+1}-\delta)-t_j^2=0 \\
\frac{\partial L}{\partial s} &= 2\lambda_{j,t}s=0 \\
\frac{\partial L}{\partial t} &= 2\mu_{j,t+1}t=0
\end{aligned}
$$

A.4 Code

Lee-Carter method

```
clear
Data = readtable("mortality_EU.csv",ReadRowNames=true,ReadVariableNames=true);
MortalityRates = Data.Variables;
Years = Data.Properties.VariableNames;
Years = categorical(Years,Years,Ordinal=true);

% Initialize the Mortality matrix
Mxt0 = Data.Variables;

% Remove year 2020 from the time series.
Mxt0 = Mxt0(:,1:50);

% Define T, the number of years
T = size(Mxt0,2);

% Calculate the estimator for the age-specific pattern of mortality coefficient,
    a_hat
a_hat = (1/T)*sum(log(Mxt0),2);

% Define matrix Z
Z = log(Mxt0)-a_hat;

% Apply SVD to Z
[U,S,V] = svd(Z);

% Solve for estimates b_hat and k_hat.
b_hat = -U(:,1);
sum_b_hat = sum(b_hat);
b_hat = b_hat/sum_b_hat;
k_hat =-sum_b_hat*S(1,1)*V(:,1);

```

```
KPSS= kpsstest(k_hat_Diff_2,'Lags',[0,1,2,3,4,5,6,7,8,9]);

A=zeros(3,5);
A=num2cell(A);
A(1,1)={[1]}; A(2,1)={[1]}; A(3,1)={[1]};
for i=1:3
    if i==1
         A(i,2)={1};
    elseif i==2
        A(i,2)={2};
    else
        A(i,2)={[1,2]};
    end
    Estimate_ARMA=estimate(arima('ARLags',A{i,1},'D',2,'MALags',A{i,2}),k_hat);
    results=summarize(Estimate_ARMA);
    A(i,3)={Estimate_ARMA.Variance}; % estimated variance
    residuals=infer(Estimate_ARMA,k_hat); %sample variance of the residuals
    A(i,4)={var(residuals)};
    A(i,5)={results.BIC};
end
colNames = {'p','q','variance','residuals','BIC'};
table2=cell(4,5);
table2(1,:)=colNames;
table2(2:end,:)=A;

% Create the ARIMA model
Mdl = arima('ARLags',1,'MALags', 1,'D',2);
Mdl = estimate(Mdl,k_hat);

summary_mdl=summarize(Mdl); % summary
Res = infer(Mdl,k_hat);      % residuals

% Forecasted mortality
Tf = 60;
[Y,YMSE] = forecast(Mdl,Tf,k_hat);

% Lower and upper 67% confidence bounds
YLower = Y-norminv(1-1/6)*sqrt(YMSE);
YUpper = Y+norminv(1-1/6)*sqrt(YMSE);

Exponent20 = a_hat + b_hat*Y(20);
mxt20 = exp(Exponent20);

Exponent40 = a_hat + b_hat*Y(40);
mxt40 = exp(Exponent40);

Exponent60 = a_hat + b_hat*Y(60);
mxt60 = exp(Exponent60);

% Compute mortality per age groups

Exponent = a_hat + b_hat.*Y';
Exponent_lower=a_hat + b_hat.*YLower';
Exponent_upper=a_hat + b_hat.*YUpper';
mxt = exp(Exponent);
mxt_low = exp(Exponent_lower);
```

```
mxt_upper = exp(Exponent_upper);
mxt2= mxt;
mxt2(92,:)=prod(1-mxt2,1);
aging_model=ones(2,Tf);
aging_low=ones(2,Tf);
aging_upper=ones(2,Tf);


AgeLimits = [0,43,67,90];

% Loop over each year
for i = 1:Tf
    % Calculate mortality rate for each age group
    for j = 2:3
        % Find rows corresponding to age group
        AgeRows = (AgeLimits(j-1)+1):AgeLimits(j);

        % Calculate mortality rate
        % Lower percentiel becomes higher percentiel because survival rate is 1-
    mortality rate
        aging_model(j-1, i) = prod(1-mxt(AgeRows,i));
        aging_upper(j-1, i) = prod(1-mxt_low(AgeRows,i));
        aging_low(j-1, i) = prod(1-mxt_upper(AgeRows,i));
    end
end

survival_2019(1,1)=prod(1-MortalityRates(1:43,50));
survival_2019(2,1)=prod(1-MortalityRates(44:67,50));
aging_model=[survival_2019,aging_model];
aging_model=aging2_model(:,[1,20,40,60]);
aging2_up=[survival_2019,aging_upper];
aging2_up=aging2_up(:,[1,20,40,60]);

colnames=['Frecasted survival rate 2','Forcasted survival rate 3','Lower 67% CI 2
    ','Lower 67% CI 3','Upper 67% CI 2', 'Upper 67% CI 3'];
results_survival=table([aging2_model(1,:)]',[aging2_model(2,:)]',[aging2_low(1,:)
    ]',[aging2_low(2,:)]',[aging2_up(1,:)]',[aging2_up(2,:)]');
writetable(results_survival, 'results_survival.csv')
```

EET tax system

```
model;
    for i=1:3
    [name='wage']
    w_i=RHOi*w;
    %% Tax related equations %%
    [name='tax rates employed agents age group 20-40']
    TAU1_i=TAU_e1+(Y_bar2<=w_i*l_1_i-s_1_i)*TAU_e2+(Y_bar3<=w_1*l_1_i-s_1_i)*
    TAU_e3;
    [name='tax rates self-employed agents age group 20-40']
    TAU1_is=TAU_e1+(Y_bar2<=w_i*l_1_is-s_1_is)*TAU_e2+(Y_bar3<=w_i*l_1_is-s_1_is)
    *TAU_e3;
    [name='tax rates employed agents age group 40-60']
    TAU2_i=TAU_e1+(Y_bar2<=w_i*l_2_i-s_2_i)*TAU_e2+(Y_bar3<=w_i*l_2_i-s_2_i)*
    TAU_e3;
```

```
[name='tax rates self-employed agents age group 40-60']
TAU2_is=TAU_e1+(Y_bar2<=w_i*l_2_is-s_2_is)*TAU_e2+(Y_bar3<=w_i*l_2_is-s_2_is)*TAU_e3;
[name='tax rates pensioners who were employed']
TAU_RETi=TAU_eRET1+(Y_bar2<=B_tilde+k_3_i(-1)*(1+r-DELTA))*TAU_eRET2+(Y_bar3<=B_tilde+k_3_i(-1)*(1+r-DELTA))*TAU_eRET3;
[name='tax rates pensioners who were self-employed']
TAU_RETis=TAU_eRET1+(Y_bar2<=B_tilde+k_3_is(-1)*(1+r-DELTA))*TAU_eRET2+(Y_bar3<=B_tilde+k_3_is(-1)*(1+r-DELTA))*TAU_eRET3;
[name='lumpsum employed agents in the age group 20-40']
T1_i=(Y_bar3>=w_i*l_1_i-s_1_i)*TAU_e1*Y_bar1+(Y_bar2<=w_i*l_1_i-s_1_i)*TAU_e2*Y_bar2+(Y_bar3<=w_i*l_1_i-s_1_i)*TAU_e3*Y_bar3;
[name='lumpsum self-employed agents age group 20-40']
T1_is=(Y_bar3>=w_i*l_1_is-s_1_is)*TAU_e1*Y_bar1+(Y_bar2<=w_i*l_1_is-s_1_is)*TAU_e2*Y_bar2+(Y_bar3<=w_i*l_1_is-s_1_is)*TAU_e3*Y_bar3;
[name='lumpsum employed agents age group 40-60']
T2_i=(Y_bar3>=w_1*l_2_i-s_2_i)*TAU_e1*Y_bar1+(Y_bar2<=w_i*l_2_i-s_2_i)*TAU_e2*Y_bar2+(Y_bar3<=w_i*l_2_i-s_2_i)*TAU_e3*Y_bar3;
[name='lumpsum selfemployed agents age group 40-60']
T2_is=(Y_bar3>=w_i*l_2_is-s_2_is)*TAU_e1*Y_bar1+(Y_bar2<=w_i*l_2_is-s_2_is)*TAU_e2*Y_bar2+(Y_bar3<=w_i*l_2_is-s_2_is)*TAU_e3*Y_bar3;
[name='lumpsum employed agents in retirement']
T_RETi=(Y_bar3>=B_tilde+k_3_i(-1)*(1+r-DELTA))*(Y_bar1<=B_tilde+k_3_i(-1)*(1+r-DELTA))*TAU_eRET1*Y_bar1+(Y_bar2<=B_tilde+k_3_i(-1)*(1+r-DELTA))*TAU_eRET2*Y_bar2+(Y_bar3<=B_tilde+k_3_i(-1)*(1+r-DELTA))*TAU_eRET3*Y_bar3;
[name='lumpsum self-employed agents in retirement']
T_RETis=(Y_bar3>=B_tilde+k_3_is(-1)*(1+r-DELTA))*(Y_bar1<=B_tilde+k_3_is(-1)*(1+r-DELTA))*TAU_eRET1*Y_bar1+(Y_bar2<=B_tilde+k_3_is(-1)*(1+r-DELTA))*TAU_eRET2*Y_bar2+(Y_bar3<=B_tilde+k_3_is(-1)*(1+r-DELTA))*TAU_eRET3*Y_bar3;
%% Savings %%
[name='savings employed']
s_1_i=sr_1*k_2_i;
s_2_i=sr_2*k_3_i-k_2_i(-1)*(1+r-DELTA);
[name='savings self-employed']
s_1_is=sr_1*k_2_is;
s_2_is=sr_2*k_3_is-k_2_is(-1)*(1+r-DELTA);
%% Budget constrants %%
[name='buget constraints for employed']
(1+TAU_C)*c_1_i=(1-TAU1_i)*(w_i*l_1_i-s_1_i)+T1_i;
(1+TAU_C)*c_2_i=(1-TAU2_i)*(w_i*l_2_i-s_2_i)+T2_i;
(1+TAU_C)*c_3_i=(1+r-DELTA)*k_3_i(-1)*(1-TAU_RET1)+B_tilde*(1-TAU_RET1)+T_RET1;
[name='buget constraints for self-employed']
(1+TAU_C)*c_1_is=(1-TAU1_is)*(w_i*l_1_is-s_1_is)+T1_is;
(1+TAU_C)*c_2_is=(1-TAU2_is)*(w_i*l_2_is-s_2_is)+T2_is;
(1+TAU_C)*c_3_is=(1+r-DELTA)*k_3_is(-1)*(1-TAU_RET1s)+B_tilde*(1-TAU_RET1s)+T_RET1s;


[name='EULER:labour-leisure choice for employed']
((c_1_i^GAMMA)*(1-l_1_i)^(1-GAMMA))^(-SIGMA)*((1-l_1_i)^(1-GAMMA)*GAMMA*(c_1_i^(GAMMA-1))*((1-TAU1_i)/(1+TAU_C))*w_i-(c_1_i^GAMMA)*(1-GAMMA)*(1-l_1_i)^(-GAMMA))+lami*PHI*w_i=0;
sr_1(-1)*BETA*((c_2_i^GAMMA)*(1-l_2_i)^(1-GAMMA))^(-SIGMA)*((1-l_2_i)^(1-GAMMA)*GAMMA*(c_2_i^(GAMMA-1))*((1-TAU2_i)/(1+TAU_C))*w_i-(c_2_i^GAMMA)*(1-GAMMA)*(1-l_2_i)^(-GAMMA))+mui*PHI*w_1=0;
```

```

 [name='EULER:labour-leisure choice for self-employed']
 ((1-TAU1_is)/(1+TAU_C))*w_i=((1-GAMMA)/GAMMA)*(c_1_is/(1-l_1_is));
 ((1-TAU2_is)/(1+TAU_C))*w_i=((1-GAMMA)/GAMMA)*(c_2_is/(1-l_2_is));

 [name='EULER: consumption-savings choice for employed']
 -(sr_1*(1-TAU1_i)/(1+TAU_C))*((1-l_1_i)^(1-GAMMA))*GAMMA*(c_1_i^(GAMMA-1))*((c_1_i^GAMMA)*(1-l_1_i)^(1-GAMMA))^(-SIGMA)+BETA*sr_1*((1-l_2_i(+1))^(1-GAMMA))*GAMMA*((c_2_i(+1))^(GAMMA-1))*(((c_2_i(+1)^GAMMA)*((1-l_2_i(+1))^(1-GAMMA)))^(-SIGMA))*((1-TAU2_i(+1))/(1+TAU_C))*(1+r(+1)-DELTA)-lami*sr_1+mui(+1)*(1+r(+1)-DELTA)=0;
 -(sr_1(-1)*BETA*(1-TAU2_i)/(1+TAU_C))*((1-l_2_i)^(1-GAMMA))*GAMMA*(c_2_i^(GAMMA-1))*((c_2_i^GAMMA)*(1-l_2_i)^(1-GAMMA))^(-SIGMA)+(BETA^2)*sr_1(-1)*GAMMA*(c_3_i(+1)^(GAMMA*(1-SIGMA)-1))*(1+r(+1)-DELTA)*((1-TAU_RETi(+1))/(1+TAU_C))-mui=0; %divided by sr_2

 s_1_i-PHI*(w_i*l_1_i-franch)-si^2=0;
 s_2_i-PHI*(w_i*l_2_i-franch)-ti^2=0;
 2*lami*si=0;
 2*mui*ti=0;

 [name='EULER: consumption-savings choice for self-employed']
 -(sr_1*(1-TAU1_is)/(1+TAU_C))*((1-l_1_is)^(1-GAMMA))*GAMMA*(c_1_is^(GAMMA-1))*((c_1_is^GAMMA)*(1-l_1_is)^(1-GAMMA))^(-SIGMA)+BETA*sr_1*((1-l_2_is(+1))^(1-GAMMA))*GAMMA*((c_2_is(+1))^(GAMMA-1))*(((c_2_is(+1)^GAMMA)*((1-l_2_is(+1))^(1-GAMMA)))^(-SIGMA))*((1-TAU2_is(+1))/(1+TAU_C))*(1+r(+1)-DELTA)-lamis*sr_1+muis(+1)*(1+r(+1)-DELTA)=0;
 -(sr_1(-1)*BETA*(1-TAU2_is)/(1+TAU_C))*((1-l_2_is)^(1-GAMMA))*GAMMA*(c_2_is^(GAMMA-1))*((c_2_is^GAMMA)*(1-l_2_is)^(1-GAMMA))^(-SIGMA)+(BETA^2)*sr_1(-1)*GAMMA*(c_3_is(+1)^(GAMMA*(1-SIGMA)-1))*(1+r(+1)-DELTA)*((1-TAU_RETis(+1))/(1+TAU_C))-muis=0; %divided by sr_2

 s_1_is-sis^2=0;
 s_2_is-tis^2=0;
 2*lamis*sis=0;
 2*muis*tis=0;

 %% certainty equivalent %%
 ceie=((1-SIGMA)*(((c_1_i^GAMMA*(1-l_1_i)^(1-GAMMA))^(1-SIGMA))/(1-SIGMA)+BETA*sr_1*((c_2_i(+1)^GAMMA*(1-l_2_i(+1))^(1-GAMMA))^(1-SIGMA))/(1-SIGMA)+BETA^2*sr_1*sr_2(+1)*((c_3_i(+2)^GAMMA)^(1-SIGMA))/(1-SIGMA)))^(1/(GAMMA*(1-SIGMA)));
 ceis=((1-SIGMA)*(((c_1_is^GAMMA*(1-l_1_is)^(1-GAMMA))^(1-SIGMA))/(1-SIGMA)+BETA*sr_1*((c_2_is(+1)^GAMMA*(1-l_2_is(+1))^(1-GAMMA))^(1-SIGMA))/(1-SIGMA)+BETA^2*sr_1*sr_2(+1)*((c_3_is(+2)^GAMMA)^(1-SIGMA))/(1-SIGMA)))^(1/(GAMMA*(1-SIGMA)));
end
```

Listing 1: First part of "EET_NL_modeqs.inc". *Note.* All equations of the different types of agents are coded as separate model equations. However, due to space considerations, I use a lope. For example, in the model I have c_1_1, c_1_2 and c_1_3, which denote the consumption at age 20-40 for the three distinct agents. c_1_is denotes the consumption of the self-employed agent of type i.

```
% FIRMS
[name='equation interest rate']
r=ALPHA*A*((K/L)^(ALPHA-1));
[name='equation real wage rate']
```

```
w=(1-ALPHA)*A*(K/L)^ALPHA;

% MARKET CLEARING
[name='market clearing: labour']
L=ratio_emp*(pop_1*(PI1*RHO1*l_1_1+PI2*RHO2*l_1_2+PI3*RHO3*l_1_3)+pop_2*(PI1*RHO1
    *l_2_1+PI2*RHO2*l_2_2+PI3*RHO3*l_2_3))+(1-ratio_emp)*(pop_1*(PI1*RHO1*l_1_1s+
    PI2*RHO2*l_1_2s+PI3*RHO3*l_1_3s)+pop_2*(PI1*RHO1*l_2_1s+PI2*RHO2*l_2_2s+PI3*
    RHO3*l_2_3s));
[name='market clearing: capital']
K=ratio_emp*(pop_2*(PI1*k_2_1(-1)+PI2*k_2_2(-1)+PI3*k_2_3(-1))+pop_3*(PI1*k_3_1
    (-1)+PI2*k_3_2(-1)+PI3*k_3_3(-1)))+(1-ratio_emp)*(pop_2*(PI1*k_2_1s(-1)+PI2*
    k_2_2s(-1)+PI3*k_2_3s(-1))+pop_3*(PI1*k_3_1s(-1)+PI2*k_3_2s(-1)+PI3*k_3_3s(-1)
    ));

% DEMOGRPHICS
sr_1=surv_2_2019 +eps_1; %eps is the total difference with the value in 2019
sr_2=surv_3_2019 +eps_2;
fert= fert_2019  +eps_3;
pop_1=pop_1_2019 +eps_4;
pop_2=pop_2_2019 +eps_5;
pop_3=pop_3_2019 +eps_6;

%%%   Not used in solving the system of nonlinear equations   %%%
[name='total consumption']
C=ratio_emp*(pop_1*(PI1*c_1_1+PI2*c_1_2+PI3*c_1_3)+pop_2*(PI1*c_2_1+PI2*c_2_2+PI3
    *c_2_3)+pop_3*(PI1*c_3_1+PI2*c_3_2+PI3*c_3_3))+(1-ratio_emp)*(pop_1*(PI1*
    c_1_1s+PI2*c_1_2s+PI3*c_1_3s)+pop_2*(PI1*c_2_1s+PI2*c_2_2s+PI3*c_2_3s)+pop_3*(
    PI1*c_3_1s+PI2*c_3_2s+PI3*c_3_3s));
[name='Total investment = Total savings under EET']
I=ratio_emp*(PI1*(pop_1*s_1_1+pop_2*s_2_1)+PI2*(pop_1*s_1_2+pop_2*s_2_2)+PI3*(
    pop_1*s_1_3+pop_2*s_2_3))+(1-ratio_emp)*(PI1*(pop_1*s_1_1s+pop_2*s_2_1s)+PI2*(
    pop_1*s_1_2s+pop_2*s_2_2s)+PI3*(pop_1*s_1_3s+pop_2*s_2_3s));
[name='output economy']
Y=C+G+K-(1-DELTA)*K(-1);
[name='governemnt spending (without state pension expenditures)']
G=ratio_emp*(PI1*(pop_1*(TAU1_1*(w_1*l_1_1-s_1_1)-T1_1)+pop_2*(TAU2_1*(w_1*l_2_1-
    s_2_1)-T2_1)+pop_3*(TAU_RET1*(k_3_1(-1)*(1+r-DELTA)+B_tilde)-T_RET1))+PI2*(
    pop_1*(TAU1_2*(w_2*l_1_2-s_1_2)-T1_2)+pop_2*(TAU2_2*(w_2*l_2_2-s_2_2)-T2_2)+
    pop_3*(TAU_RET2*(k_3_2(-1)*(1+r-DELTA)+B_tilde)-T_RET2))+PI3*(pop_1*(TAU1_3*(
    w_3*l_1_3-s_1_3)-T1_3)+pop_2*(TAU2_3*(w_3*l_2_3-s_2_3)-T2_3)+pop_3*(TAU_RET3*(
    k_3_3(-1)*(1+r-DELTA)+B_tilde)-T_RET3)))+(1-ratio_emp)*(PI1*(pop_1*(TAU1_1s*(
    w_1*l_1_1s-s_1_1s)-T1_1s)+pop_2*(TAU2_1s*(w_1*l_2_1s-s_2_1s)-T2_1s)+pop_3*(
    TAU_RET1s*(k_3_1s(-1)*(1+r-DELTA)+B_tilde)-T_RET1s))+PI2*(pop_1*(TAU1_2s*(w_2*
    l_1_2s-s_1_2s)-T1_2s)+pop_2*(TAU2_2s*(w_2*l_2_2s-s_2_2s)-T2_2s)+pop_3*(
    TAU_RET2s*(k_3_2s(-1)*(1+r-DELTA)+B_tilde)-T_RET2s))+PI3*(pop_1*(TAU1_3s*(w_3*
    l_1_3s-s_1_3s)-T1_3s)+pop_2*(TAU2_3s*(w_3*l_2_3s-s_2_3s)-T2_3s)+pop_3*(
    TAU_RET3s*(k_3_3s(-1)*(1+r-DELTA)+B_tilde)-T_RET3s))) + TAU_C*C-pop_3*B_tilde;
[name='tax revenue']
T=G+pop_3*B_tilde;

% used for calibration
l=ratio_emp*(PI1*((l_1_1+l_2_1)/2)+PI2*((l_1_2+l_2_2)/2)+PI3*((l_1_3+l_2_3)/2))
    +(1-ratio_emp)*(PI1*((l_1_1s+l_2_1s)/2)+PI2*((l_1_2s+l_2_2s)/2)+PI3*((l_1_3s+
    l_2_3s)/2));
l_1=(l_1_1+l_2_1)/2;
l_2=(l_1_2+l_2_2)/2;
```

```
l_3=(l_1_3+l_2_3)/2;
l_1s=(l_1_1s+l_2_1s)/2;
l_2s=(l_1_2s+l_2_2s)/2;
l_3s=(l_1_3s+l_2_3s)/2;
salary1=w_1*(ratio_emp*l_1+(1-ratio_emp)*l_1s);
salary2=w_2*(ratio_emp*l_2+(1-ratio_emp)*l_2s);
salary3=w_3*(ratio_emp*l_3+(1-ratio_emp)*l_3s);

% check
Y2=A*L^(1-ALPHA)*K^ALPHA;
end;
```

Listing 2: Second part of "EET_NL_modeqs.inc". Last model equations, including market clearing conditions.

```
%% Endogeneous Variables
var
;
%% Exogenous Variables
varexo
    eps_1
    eps_2
    eps_3
    eps_4
    eps_5
    eps_6
;
%% Declaration of Parameters
parameters
;
```

Listing 3: "EET_NL_symdecl.inc" file. *Note.* The set-up for the symbol declaration is similar for EET and TEE. I will only show the set-up due to space considerations.

```

@#define sensitivity=0

@#include "EET_NL_symdecl.inc"

% Preferences and technology
ALPHA=0.25;  A=1;
DELTA=1-(1-0.0718)^20; BETA=0.99^20; SIGMA=1.5;

% Tax system
TAU_C=0.21;           TAU1=0.3735;           TAU2=0.3735;                TAU3
    =0.4950;
TAU_e1=0.3735;        TAU_e2=0;              TAU_e3=0.4950-0.3735;
TAU_eRET1=0.1945;     TAU_eRET2=0.3735-0.1945; TAU_eRET3=0.4950-0.3735;
Y_bar1=7100/(100000);  Y_bar2=34712/(100000);   Y_bar3=68507/(100000);

% Demographics
fert_2019=0.785;      ratio_emp=1-0.195;
surv_2_2019=0.98045;  surv_3_2019=0.89355;
pop_1_2019=0.35007;   pop_2_2019=0.35007*surv_2_2019;    pop_3_2019=0.35007*
    surv_2_2019*surv_3_2019;
PI1=0.3205;           PI2=0.3708;          PI3=0.3088;
```

```
% pension
B_tilde=12232/100000;   PHI=0.2;            franch=16310/100000;

%initial guess
GAMMA=0.36;
RHO1=24.552/(100*0.424251*1/3);     RHO2=50.990/(100*0.424251*1/3);     RHO3
   =90.140/(100*0.424251*1/3);

@#if sensitivity == 0
    %initial guess
    GAMMA=0.36;
    RHO1=24.552/(100*0.424251*1/3);     RHO2=50.990/(100*0.424251*1/3);     RHO3
   =90.140/(100*0.424251*1/3);
@#elseif sensitivity == 1
    GAMMA=0.329005;%calibrated in "EET_steady2"
    RHO1=1.73737;
    RHO2=3.32047;
    RHO3=6.33339;
    l_2_3=0.28;
    l_1_3s=0.28;
    l_2_3s=0.28;
@#elseif sensitivity == 2
    GAMMA=0.329005;%calibrated in "EET_steady2"
    RHO1=1.73737;
    RHO2=3.32047;
    RHO3=6.33339;
    SIGMA=1.1*1.5;
@#elseif sensitivity == 3
    GAMMA=0.329005;%calibrated in "EET_steady2"
    RHO1=1.73737;
    RHO2=3.32047;
    RHO3=6.33339;
    SIGMA=0.9*1.5;
@#elseif sensitivity == 4
    GAMMA=0.329005;%calibrated in "EET_steady2"
    RHO1=1.73737;
    RHO2=3.32047;
    RHO3=6.33339;
    ratio_emp=1;
@#elseif sensitivity == 5
    GAMMA=0.329005;%calibrated in "EET_steady2"
    RHO1=1.73737;
    RHO2=3.32047;
    RHO3=6.33339;
    A=1.1;
@#endif

@#include "EET_NL_modeqs.inc"

initval;
%% initial value as starting point for the Newton algorithm
end;

resid;
steady;
```

```
@#if sensitivity == 0
    save_params_and_steady_state('EET_NL_steady1.txt');
@#elseif sensitivity == 1
    writecell([M_.endo_names,mat2cell(oo_.steady_state,[ones(1,202)],1)],'EET_alpha10%.txt');
@#elseif sensitivity == 2
    writecell([M_.endo_names,mat2cell(oo_.steady_state,[ones(1,206)],1)],'EET_sigma10%.txt');
@#elseif sensitivity == 3
    writecell([M_.endo_names,mat2cell(oo_.steady_state,[ones(1,206)],1)],'EET_bas_sigma_low.txt');
@#elseif sensitivity == 4
    writecell([M_.endo_names,mat2cell(oo_.steady_state,[ones(1,206)],1)],'EET_verplichtpensioen.txt');
@#elseif sensitivity == 5
    writecell([M_.endo_names,mat2cell(oo_.steady_state,[ones(1,206)],1)],'EET_A10%.txt');
@#endif
```

Listing 4: "EET_steady1.mod" file

```
@#define STEADY=1

@#include "EET_NL_symdecl.inc"
change_type(parameters) l salary1 salary2 salary3; %w
change_type(var)  GAMMA RHO1 RHO2 RHO3; %A

@#include "EET_NL_modeqs.inc"

load_params_and_steady_state('EET_NL_steady1.txt');

set_param_value('salary1',0.24552);
set_param_value('salary2',0.50990);
set_param_value('salary3',0.90140);
set_param_value('l',1/3);

steady;
save_params_and_steady_state('EET_steady2.txt');
```

Listing 5: "EET_steady2.mod" file. Calibration method of modularization and changing types, to set l and the average salaries equal to the targets.

```
@#define STEADY=1
@#define SCENARIO=2

@#include "EET_NL_symdecl.inc"
@#include "EET_NL_modeqs.inc"

load_params_and_steady_state('EET_steady2.txt');
resid;
steady;

%% deterministic simulation %%
@#if SCENARIO == 1
    shocks;
```

```
    var eps_1; periods 1 2  3;
    values 0.00453106274036896 0.00818211795880997 0.0110498281375870;
    var eps_2; periods 1  2 3;
    values 0.0177682405445430 0.0352858491737450 0.0503187787808199;
    var eps_3; periods  1 2 3;
    values 0.0650000000000000 0.0650000000000000  0.0650000000000000;
    var eps_4; periods 1 2 3;
    values 0.0474895000000000 0.0578555750000000  0.0666667387500000;
    var eps_5; periods 1 2 3;
    values 1.31995821472941e-06 0.0483639463325196  0.0600637317876619;
    var eps_6; periods 1 2 3;
    values -7.77411706731623e-07  0.00609894995296628 0.0570323057130743;
    end;
    perfect_foresight_simulation;
    perfect_foresight_setup(periods=3);
    perfect_foresight_solver(maxit=100);
    writecell([M_.endo_names,mat2cell(oo_.endo_simul,[ones(1,206)],[ones(1,5)])],
   'Results_bas_EET.txt')
 @#elseif SCENARIO == 2
    shocks;
    var eps_1; periods 1 2  3;
    values 0.00712665010133096 0.0125481832936790 0.0158822102708160;
    var eps_2; periods 1  2 3;
    values 0.0300533502132060 0.0587289032354280 0.0792378078389600;
    var eps_3; periods  1 2 3;
    values 0.0650000000000000 0.0650000000000000  0.0650000000000000;
    var eps_4; periods 1 2 3;
    values 0.0474895000000000 0.0578555750000000  0.0666667387500000;
    var eps_5; periods 1 2 3;
    values 1.31995821472941e-06 0.0493958467459499  0.0618447614998760;
    var eps_6; periods 1 2 3;
    values -7.77411706731623e-07  0.0103155368354262  0.0671950277344023;
    end;
    perfect_foresight_simulation
    perfect_foresight_setup(periods=3);
    perfect_foresight_solver(maxit=100);
    writecell([M_.endo_names,mat2cell(oo_.endo_simul,[ones(1,206)],[ones(1,5)])],
   'Results_EET_sr_high.txt')
@#elseif SCENARIO == 3
    shocks;
    var eps_1; periods 1 2  3;
    values 0.00453106274036896 0.00818211795880997 0.0110498281375870;
    var eps_2; periods 1  2 3;
    values 0.0177682405445430 0.0352858491737450 0.0503187787808199;
    var eps_3; periods  1 2 3;
    values -0.0200000000000000  -0.0550000000000001 -0.0550000000000001;
    var eps_4; periods 1 2 3;
    values 0.0177335500000000 -0.0115734085000000 -0.0329674882050000;
    var eps_5; periods 1 2 3;
    values 1.31995821472941e-06 0.0190547868822221  -0.00857625300347631;
    var eps_6; periods 1 2 3;
    values -7.77411706731623e-07  0.00609894995296628 0.0298089740921205;
    end;
    perfect_foresight_simulation
    perfect_foresight_setup(periods=3);
    perfect_foresight_solver(maxit=100);
```

```
    writecell([M_.endo_names,mat2cell(oo_.endo_simul,[ones(1,206)],[ones(1,5)])], 'Results_EET_fr_low.txt')
@#endif
```

Listing 6: "EET_steady3.mod" file. Perfect-foresight simulations in the three scenarios. *Note.* In the perfect foresight simulations, I use the parameter values set in "EET_steady2.mod".

TEE tax system

```
model;
for i=1:3
    %wage
    w_i=RHOi*w;

    [name='tax rates employed agents age group 20-40']
    TAU1_i=TAU_e1+(Y_bar2<=w_i*l_1_i)*TAU_e2+(Y_bar3<=w_1*l_1_i)*TAU_e3;
    [name='tax rates self-employed agents age group 20-40']
    TAU1_is=TAU_e1+(Y_bar2<=w_i*l_1_is)*TAU_e2+(Y_bar3<=w_1*l_1_is)*TAU_e3;

    [name='tax rates employed agents age group 40-60']
    TAU2_i=TAU_e1+(Y_bar2<=w_1*l_2_i)*TAU_e2+(Y_bar3<=w_i*l_2_i)*TAU_e3;
    [name='tax rates self-employed agents age group 40-60']
    TAU2_is=TAU_e1+(Y_bar2<=w_1*l_2_is)*TAU_e2+(Y_bar3<=w_i*l_2_is)*TAU_e3;
    [name='lumpsum employed agents in the age group 20-40']
    T1_i=(Y_bar3>=w_i*l_1_i)*TAU_e1*Y_bar1+(Y_bar2<=w_i*l_1_i)*TAU_e2*Y_bar2+(Y_bar3<=w_i*l_1_i)*TAU_e3*Y_bar3;
    [name='lumpsum self-employed agents age group 20-40']
    T1_is=(Y_bar3>=w_1*l_1_is)*TAU_e1*Y_bar1+(Y_bar2<=w_i*l_1_is)*TAU_e2*Y_bar2+(Y_bar3<=w_i*l_1_is)*TAU_e3*Y_bar3;
    [name='lumpsum employed agents age group 40-60']
    T2_i=(Y_bar3>=w_i*l_2_i)*TAU_e1*Y_bar1+(Y_bar2<=w_i*l_2_i)*TAU_e2*Y_bar2+(Y_bar3<=w_i*l_2_i)*TAU_e3*Y_bar3;
    [name='lumpsum selfemployed agents age group 40-60']
    T2_is=(Y_bar3>=w_i*l_2_is)*TAU_e1*Y_bar1+(Y_bar2<=w_i*l_2_is)*TAU_e2*Y_bar2+(Y_bar3<=w_i*l_2_is)*TAU_e3*Y_bar3;
    [name='lumpsum employed agents in retirement']
    T_RETi=(Y_bar3>=B_tilde+(1+r-DELTA)*k_3_i(-1))*TAU_RET*Y_bar1;
    [name='lumpsum self-employed agents in retirement']
    T_RETis=(Y_bar3>=B_tilde+(1+r-DELTA)*k_3_is(-1))*TAU_RET*Y_bar1;

    [name='savings']
    s_1_i=(sr_1*k_2_i)/(1+TAU_b);
    s_2_i=(sr_2*k_3_i-k_2_i(-1)*(1+r-DELTA))/(1+TAU_b);

    [name='savings self-employed']
    s_1_is=(sr_1*k_2_is)/(1+TAU_b);
    s_2_is=(sr_2*k_3_is-k_2_is(-1)*(1+r-DELTA))/(1+TAU_b);

    [name='buget constraints for j=1']
    (1+TAU_C)*c_1_i+s_1_i=(1-TAU1_i)*w_1*l_1_i+T1_i;
    (1+TAU_C)*c_2_i+s_2_i=(1-TAU2_i)*w_1*l_2_i+T2_i;
    (1+TAU_C)*c_3_i=(1+r-DELTA)*k_3_i(-1)+B_tilde*(1-TAU_RETi)+T_RETi;

    [name='buget constraints for j=1 self-employed']
    (1+TAU_C)*c_1_is+s_1_is=(1-TAU1_is)*w_i*l_1_is+T1_is;
    (1+TAU_C)*c_2_is+s_2_is=(1-TAU2_is)*w_i*l_2_is+T2_is;
```

```
(1+TAU_C)*c_3_is=(1+r-DELTA)*k_3_1s(-1)+B_tilde*(1-TAU_RETi)+T_RETis;

[name='EULER:labour-leisure choice for j=1']
((c_1_i^GAMMA)*(1-l_1_i)^(1-GAMMA))^(-SIGMA)*((1-l_1_i)^(1-GAMMA)*GAMMA*(c_1_i^(GAMMA-1))*((1-TAU1_i)/(1+TAU_C))*w_i-(c_1_i^GAMMA)*(1-GAMMA)*(1-l_1_i)^(-GAMMA))+lami*PHI*w_i*(1-TAU_RETi)=0;
sr_1(-1)*BETA*((c_2_i^GAMMA)*(1-l_2_i)^(1-GAMMA))^(-SIGMA)*((1-l_2_i)^(1-GAMMA)*GAMMA*(c_2_i^(GAMMA-1))*((1-TAU2_i)/(1+TAU_C))*w_i-(c_2_i^GAMMA)*(1-GAMMA)*(1-l_2_i)^(-GAMMA))+mu1*PHI*w_i*(1-TAU_RETi)=0;

[name='EULER:labour-leisure choice for j=1 self-employed']
((1-TAU1_is)/(1+TAU_C))*w_i=((1-GAMMA)/GAMMA)*(c_1_is/(1-l_1_is));
((1-TAU2_is)/(1+TAU_C))*w_i=((1-GAMMA)/GAMMA)*(c_2_is/(1-l_2_is));

[name='EULER: consumption-savings choice for j=1']
-(((c_1_i^GAMMA)*(1-l_1_i)^(1-GAMMA))^(-SIGMA))*GAMMA*((c_1_i)^(GAMMA-1))*((1-l_1_i)^(1-GAMMA))*(sr_1/((1+TAU_b)*(1+TAU_C)))+BETA*sr_1*(((c_2_i(+1)^GAMMA)*(1-l_2_i(+1))^(1-GAMMA))^(-SIGMA))*GAMMA*(c_2_i(+1)^(GAMMA-1))*((1-l_2_i(+1))^(1-GAMMA))*(1/((1+TAU_b)*(1+TAU_C)))*(1+r(+1)+DELTA)-lami*sr_1+mui(+1)*(1+r(+1)-DELTA)=0;
-BETA*sr_1(-1)*(((c_2_i^GAMMA)*(1-l_2_i)^(1-GAMMA))^(-SIGMA))*GAMMA*(c_2_i^(GAMMA-1))*((1-l_2_i)^(1-GAMMA))*(sr_2/((1+TAU_b)*(1+TAU_C)))+BETA^2*sr_1(-1)*sr_2*GAMMA*(c_3_i(+1)^(GAMMA*(1-SIGMA)-1))*(1/(1+TAU_C))*(1+r(+1)-DELTA)-mui*sr_2=0;

s_1_i*(1+TAU_b)-PHI*(w_i*l_1_i-franch)*(1-TAU_RETi)-si^2=0;
s_2_i*(1+TAU_b)-PHI*(w_i*l_2_i-franch)*(1-TAU_RETi)-ti^2=0;
2*lami*si=0;
2*mui*ti=0;

[name='EULER: consumption-savings choice for j=1 self-employed']
-(((c_1_is^GAMMA)*(1-l_1_is)^(1-GAMMA))^(-SIGMA))*GAMMA*((c_1_is)^(GAMMA-1))*((1-l_1_is)^(1-GAMMA))*(sr_1/((1+TAU_b)*(1+TAU_C)))+BETA*sr_1*(((c_2_is(+1)^GAMMA)*(1-l_2_is(+1))^(1-GAMMA))^(-SIGMA))*GAMMA*(c_2_is(+1)^(GAMMA-1))*((1-l_2_is(+1))^(1-GAMMA))*(1/((1+TAU_b)*(1+TAU_C)))*(1+r(+1)+DELTA)-lamis*sr_1+muis(+1)*(1+r(+1)-DELTA)=0;
-BETA*sr_1(-1)*(((c_2_is^GAMMA)*(1-l_2_is)^(1-GAMMA))^(-SIGMA))*GAMMA*((c_2_is)^(GAMMA-1))*((1-l_2_is)^(1-GAMMA))*(sr_2/((1+TAU_b)*(1+TAU_C)))+BETA^2*sr_1(-1)*sr_2*GAMMA*(c_3_is(+1)^(GAMMA*(1-SIGMA)-1))*(1/(1+TAU_C))*(1+r(+1)-DELTA)-muis*sr_2=0;

s_1_is*(1+TAU_b)-sis^2=0;
s_2_is*(1+TAU_b)-tis^2=0;
2*lamis*sis=0;
2*muis*tis=0;
% certainty equivalence
ceie=((1-SIGMA)*(((c_1_i^GAMMA*(1-l_1_i)^(1-GAMMA))^(1-SIGMA))/(1-SIGMA)+BETA*sr_1*((c_2_i(+1)^GAMMA*(1-l_2_i(+1))^(1-GAMMA))^(1-SIGMA))/(1-SIGMA)+BETA^2*sr_1*sr_2(+1)*((c_3_i(+2)^GAMMA)^(1-SIGMA))/(1-SIGMA)))^(1/(GAMMA*(1-SIGMA)));
ceis=((1-SIGMA)*(((c_1_is^GAMMA*(1-l_1_is)^(1-GAMMA))^(1-SIGMA))/(1-SIGMA)+BETA*sr_1*((c_2_is(+1)^GAMMA*(1-l_2_is(+1))^(1-GAMMA))^(1-SIGMA))/(1-SIGMA)+BETA^2*sr_1*sr_2(+1)*((c_3_is(+2)^GAMMA)^(1-SIGMA))/(1-SIGMA)))^(1/(GAMMA*(1-SIGMA)));

```

```
end
```

Listing 7: First part of "TEE_NL_modeqs.inc". *Note.* All equations of the different types of agents are coded as separate model equations. However, due to space considerations, I use a lope. For example, in the model I have c_1_1, c_1_2 and c_1_3, which denote the consumption at age 20-40 for the three distinct agents. c_1_is denotes the consumption of the self-employed agent of type i.

```
% FIRMS
[name='equation interest rate']
r=ALPHA*A*((K/L)^(ALPHA-1));
[name='equation real wage rate']
w=(1-ALPHA)*A*(K/L)^ALPHA;

% DEMOGRPHICS
sr_1=surv_2_2019 +eps_1; %eps is the total difference with the value in 2019
sr_2=surv_3_2019 +eps_2;
fert=fert_2019   +eps_3;
pop_1=pop_1_2019 +eps_4;
pop_2=pop_2_2019 +eps_5;
pop_3=pop_3_2019 +eps_6;

% MARKET CLEARING
[name='market clearing: labour']
L=ratio_emp*(pop_1*(PI1*RHO1*l_1_1+PI2*RHO2*l_1_2+PI3*RHO3*l_1_3)+pop_2*(PI1*RHO1*l_2_1+PI2*RHO2*l_2_2+PI3*RHO3*l_2_3))+(1-ratio_emp)*(pop_1*(PI1*RHO1*l_1_1s+PI2*RHO2*l_1_2s+PI3*RHO3*l_1_3s)+pop_2*(PI1*RHO1*l_2_1s+PI2*RHO2*l_2_2s+PI3*RHO3*l_2_3s));
[name='market clearing: capital']
K=ratio_emp*(pop_2*(PI1*k_2_1(-1)+PI2*k_2_2(-1)+PI3*k_2_3(-1))+pop_3*(PI1*k_3_1(-1)+PI2*k_3_2(-1)+PI3*k_3_3(-1)))+(1-ratio_emp)*(pop_2*(PI1*k_2_1s(-1)+PI2*k_2_2s(-1)+PI3*k_2_3s(-1))+pop_3*(PI1*k_3_1s(-1)+PI2*k_3_2s(-1)+PI3*k_3_3s(-1)));

%%%  Not used in solving the system of nonlinear equations  %%%
[name='total consumption']
C=ratio_emp*(pop_1*(PI1*c_1_1+PI2*c_1_2+PI3*c_1_3)+pop_2*(PI1*c_2_1+PI2*c_2_2+PI3*c_2_3)+pop_3*(PI1*c_3_1+PI2*c_3_2+PI3*c_3_3))+(1-ratio_emp)*(pop_1*(PI1*c_1_1s+PI2*c_1_2s+PI3*c_1_3s)+pop_2*(PI1*c_2_1s+PI2*c_2_2s+PI3*c_2_3s)+pop_3*(PI1*c_3_1s+PI2*c_3_2s+PI3*c_3_3s));
[name='total savings']
S=ratio_emp*(pop_1*(PI1*s_1_1+PI2*s_1_2+PI3*s_1_3)+pop_2*(PI1*s_2_1+PI2*s_2_2+PI3*s_2_3))+(1-ratio_emp)*(pop_1*(PI1*s_1_1s+PI2*s_1_2s+PI3*s_1_3s)+pop_2*(PI1*s_2_1s+PI2*s_2_2s+PI3*s_2_3s));
[name='total investment']
I=(1+TAU_b)*S;
[name='output economy']
Y=C+G+K-(1-DELTA)*K(-1);
[name='governemnt spending (without state pension  and subsidy expenditures)']
G=ratio_emp*(PI1*(pop_1*(TAU1_1*w_1*l_1_1-T1_1)+pop_2*(TAU2_1*w_1*l_2_1-T2_1)+pop_3*(TAU_RET*B_tilde-T_RET1))+PI2*(pop_1*(TAU1_2*w_2*l_1_2-T1_2)+pop_2*(TAU2_2*w_2*l_2_2-T2_2)+pop_3*(TAU_RET*B_tilde-T_RET2))+PI3*(pop_1*(TAU1_3*w_3*l_1_3-T1_3)+pop_2*(TAU2_3*w_3*l_2_3-T2_3)+pop_3*(TAU_RET*(B_tilde)-T_RET3)))+(1-ratio_emp)*(PI1*(pop_1*(TAU1_1s*w_1*l_1_1s-T1_1s)+pop_2*(TAU2_1s*w_1*l_2_1s-T2_1s)+pop_3*(TAU_RET*(B_tilde)-T_RET1s))+PI2*(pop_1*(TAU1_2s*w_2*l_1_2s-T1_2s)+pop_2*(TAU2_2s*w_2*l_2_2s-T2_2s)+pop_3*(TAU_RET*(B_tilde)-T_RET2s))+PI3*(pop_1*(TAU1_3s*w_3*l_1_3s-T1_3s)+pop_2*(TAU2_3s*w_3*l_2_3s-
```

```
    T2_3s)+pop_3*(TAU_RET*(B_tilde)-T_RET3s)))-TAU_b*S +TAU_C*C-pop_3*B_tilde;
[name='net tax revenue']
T=G+pop_3*B_tilde;

% used in the sensitivity analyses
le=PI1*l_1+PI2*l_2+PI3*l_3;
ls=PI1*l_1s+PI2*l_2s+PI3*l_3s;
l=ratio_emp*le+(1-ratio_emp)*ls;

%check
Y2=A*L^(1-ALPHA)*K^ALPHA;
@#endif
end;
```

Listing 8: Second part of "TEE_NL_modeqs.inc". Last model equations, including market clearing conditions.

```
=%addpath("C:\Users\Chris\Documents\Master_thesis_matlab\5.4\matlab")

@#define SCENARIO = 0
@#define SCENARIOb = 0
@#define sensitivity = 0
@#include "TEE_NL_symdecl.inc"

% Preferences and technology
ALPHA=0.25;  A=1;
DELTA=1-(1-0.0718)^20; BETA=0.99^20; SIGMA=1.5;

% Calibrated under the OLG model with the EET tax system
GAMMA=0.329005;   RHO1=1.73737;   RHO2=3.32047;    RHO3=6.33339;

% Tax system
TAU_C=0.21;     TAU1=0.3735;      TAU2=0.3735;             TAU3=0.4950;
TAU_e1=0.3735;  TAU_e2=0;         TAU_e3=0.4950-0.3735;    TAU_eRET=0.1945;
Y_bar1=7100/(100000);  Y_bar2=34712/(100000); Y_bar3=68507/(100000);
 TAU_RET2=0.3735; TAU_RET3=0.495; %TAU_RET1=TAU_eRET;

% Demographics
fert_2019=0.785;      ratio_emp=1-0.195;
surv_2_2019=0.98045;  surv_3_2019=0.89355;
pop_1_2019=0.35007;      pop_2_2019=0.35007*surv_2_2019;    pop_3_2019=0.35007*
    surv_2_2019*surv_3_2019;
PI1=0.3205;           PI2=0.3708;          PI3=0.3088;

% pension
B_tilde=12232/100000;  PHI=0.2;   franch=16310/100000;

@#if SCENARIOb == 1
    TAU_b=0;
@#elseif SCENARIOb == 2
    TAU_b=0.1;
@#elseif SCENARIOb == 3
    TAU_b=0.2;
@#elseif SCENARIOb == 4
    TAU_b=0.3;
@#elseif sensitivity == 1
```

```
    ALPHA=1.1*(1-0.75);
    TAU_b=0.2;
    l_1_3s=0.342;
    l_2_3s=0.342;
    l_2_3=0.342;
@#elseif sensitivity == 2
    SIGMA=1.1*1.5;
    TAU_b=0.2;
@#elseif sensitivity == 3
    SIGMA=0.9*1.5;
    TAU_b=0.2;
@#elseif sensitivity == 4
    TAU_b=0.2;
    ratio_emp=1;
@#elseif sensitivity == 5
    TAU_b=0.2;
    A=1.1;
@#elseif sensitivity == 6
    TAU_b=0.2;
@#elseif SCENARIO == 1
    TAU_b=0.2;
@#elseif SCENARIO == 2
    TAU_b=0.2;
@#elseif SCENARIO == 3
    TAU_b=0.2;
@#endif

@#include "TEE_NL_modeqs.inc"

%@#include "initval10.inc"

initval;
%% initial starting point for the Newton algorithm
end;

resid;
steady(maxit=1000);

save_params_and_steady_state('TEE_steady_final.txt');
save_params_and_steady_state('TEE_steady_vergelijken.txt');

check;
@#if SCENARIOb == 1
    writecell([M_.endo_names, mat2cell(oo_.steady_state,[ones(1,209)],1)],'
   Results_thesis_TEE_0.txt')
@#elseif SCENARIOb == 2
    writecell([M_.endo_names, mat2cell(oo_.steady_state,[ones(1,209)],1)],'
   Results_thesis_TEE_10.txt')
@#elseif SCENARIOb == 3
    writecell([M_.endo_names, mat2cell(oo_.steady_state,[ones(1,209)],1)],'
   Results_thesis_TEE_20.txt')
@#elseif SCENARIOb == 4
    writecell([M_.endo_names, mat2cell(oo_.steady_state,[ones(1,209)],1)],'
   Results_thesis_TEE_30.txt')
@#elseif sensitivity == 1
    writecell([M_.endo_names, mat2cell(oo_.steady_state,[ones(1,206)],1)],'
```

```
TEE_alpha10%.txt')
 @#elseif sensitivity == 2
    writecell([M_.endo_names, mat2cell(oo_.steady_state,[ones(1,209)],1)],'
   TEE_sigma10%.txt')
 @#elseif sensitivity == 3
    writecell([M_.endo_names, mat2cell(oo_.steady_state,[ones(1,209)],1)],'
   TEE_sigma_low.txt')
 @#elseif sensitivity == 4
    writecell([M_.endo_names, mat2cell(oo_.steady_state,[ones(1,209)],1)],'
   TEE_verplichtpensioen.txt')
 @#elseif sensitivity == 5
    writecell([M_.endo_names, mat2cell(oo_.steady_state,[ones(1,209)],1)],'
   TEE_A10%.txt')
 @#elseif sensitivity == 6
     save_params_and_steady_state('TEE_sensitivity6.txt');
@#endif

%% deterministic simulation %%
@#if SCENARIO == 1
    TAU_b=0.2;
    shocks;
    var eps_1; periods 1 2  3;
    values 0.00453106274036896 0.00818211795880997 0.0110498281375870;
    var eps_2; periods 1  2 3;
    values 0.0177682405445430 0.0352858491737450 0.0503187787808199;
    var eps_3; periods  1 2 3;
    values 0.0650000000000000 0.0650000000000000  0.0650000000000000;
    var eps_4; periods 1 2 3;
    values 0.0474895000000000 0.0578555750000000  0.0666667387500000;
    var eps_5; periods 1 2 3;
    values 1.31995821472941e-06 0.0483639463325196  0.0600637317876619;
    var eps_6; periods 1 2 3;
    values -7.77411706731623e-07  0.00609894995296628 0.0570323057130743;
    end;

    perfect_foresight_simulation
    perfect_foresight_setup(periods=3);
    perfect_foresight_solver(maxit=1000);
    writecell([M_.endo_names,mat2cell(oo_.endo_simul,[ones(1,210)],[ones(1,5)])],
   'TEE_bas_20.txt')
 @#elseif SCENARIO == 2
    TAU_b=0.2;
    shocks;
    var eps_1; periods 1 2  3;
    values 0.00712665010133096 0.0125481832936790 0.0158822102708160;
    var eps_2; periods 1  2 3;
    values 0.0300533502132060 0.0587289032354280 0.0792378078389600;
    var eps_3; periods  1 2 3;
    values 0.0650000000000000 0.0650000000000000  0.0650000000000000;
    var eps_4; periods 1 2 3;
    values 0.0474895000000000 0.0578555750000000  0.0666667387500000;
    var eps_5; periods 1 2 3;
    values 1.31995821472941e-06 0.0493958467459499  0.0618447614998760;
    var eps_6; periods 1 2 3;
    values -7.77411706731623e-07  0.0103155368354262  0.0671950277344023;
    end;
```

```

    perfect_foresight_simulation
    perfect_foresight_setup(periods=3);
    perfect_foresight_solver(maxit=1000);
    writecell([M_.endo_names,mat2cell(oo_.endo_simul,[ones(1,210)],[ones(1,5)])],'TEE_endo_20_sr_high.txt')
@#elseif SCENARIO == 3
    TAU_b=0.2;
    shocks;
    var eps_1; periods 1 2  3;
    values 0.00453106274036896 0.00818211795880997 0.0110498281375870;
    var eps_2; periods 1  2 3;
    values 0.0177682405445430 0.0352858491737450 0.0503187787808199;
    var eps_3; periods  1 2 3;
    values -0.0200000000000000  -0.0550000000000001 -0.0550000000000001;
    var eps_4; periods 1 2 3;
    values 0.0177335500000000 -0.0115734085000000 -0.0329674882050000;
    var eps_5; periods 1 2 3;
    values 1.31995821472941e-06 0.0190547868822221  -0.00857625300347631;
    var eps_6; periods 1 2 3;
    values -7.77411706731623e-07  0.00609894995296628 0.0298089740921205;
    end;

    perfect_foresight_simulation
    perfect_foresight_setup(periods=3);
    perfect_foresight_solver(maxit=100);
    writecell([M_.endo_names,mat2cell(oo_.endo_simul,[ones(1,210)],[ones(1,5)])],'TEE_endo_20_fr_low.txt')
@#endif
```

Listing 9: "TEE_steady1.mod" file Calculating both the steady-state simulations and perfect-foresight simulations.

```
@#include "TEE_NL_symdecl.inc"
change_type(parameters) l;
change_type(var)  GAMMA;

@#include "TEE_NL_modeqs.inc"

load_params_and_steady_state('TEE_sensitivity6.txt');

l_2_3s=0.268;
set_param_value('l',1/3);

steady;
writecell([M_.endo_names, mat2cell(oo_.steady_state,[ones(1,209)],1)],'TEE_fixedlabour.txt')
```

Listing 10: "TEE_steady2.mod" file. Calibration method of modularization and changing types, to set l under TEE equal to the target: for the sensitivity analysis.